Japan Concrete Institute Standard

Method of test for bending moment–curvature curve of fiber-reinforced cementitious composites

JCI-S-003-2007

1. Scope
This specification specifies the test method for bending moment–curvature curve of fiber-reinforced cementitious composites (FRCC)\(^{(1)}\) that show separated plural cracks\(^{(2)}\) under pure bending stress by 4-point bending test.

Note (1): The length of a fiber to be covered by this standard is 40mm or less.
Note (2): Separated plural cracks mean two or more independent cracks visible to the eye occurring in the pure bending span before maximum load is observed, as shown in Figure 1.

![Cracking covered by this standard](image)

Figure 1  Cracking covered by this standard

Remark: Tensile strength and ultimate tensile strain can be evaluated by the Appendix of JCI-S-003-2007 using maximum bending moment and curvature obtained by this test method.

2. Normative references
The following normative documents contain provisions which, through reference in this text, constitute a portion of the provisions of JCI-S-003-2007. The latest editions of these citations shall apply.

JIS A 1132: Method of making and curing concrete specimens
JIS A 1106: Method of test for flexural strength of concrete

3. Test specimens
3.1 Dimensions of test specimens
The test specimen shall be a beam having a square cross section 100mm in depth (\(D\)), 100mm in width (\(B\)) and 400mm in length (\(L\)).

3.2 Molding of test specimens
a) The mold specified by JIS A 1132 (Method of making and curing concrete specimens) shall be used.
b) FRCC shall be placed in one placing sequence without joints, with attention being paid to fiber orientation. In no case shall FRCC be consolidated using a tamping rod or internally vibrated by inserting a vibrator head.

3.3 Preparation of hardened test specimens
a) The depth, width, length and mass of each test specimen shall be measured.
b) The apparent density of each test specimen shall be calculated by dividing its measured mass by the volume calculated from its measured dimensions.

4. Test equipment
4.1 4-point loading equipment
The loading equipment specified by JIS A 1106 “Method of test for flexural strength of concrete” shall be used. The span length shall be 300mm.

4.2 Load measuring equipment
The load shall be measured by a load cell having an accuracy within 1% of the maximum load. The load cell shall be set to the loading equipment.

4.3 Curvature measuring equipment

a) The curvature measuring equipment shall consist of linear variable displacement transducers (LVDTs) and jigs used for fixing LVDTs. LVDTs having an accuracy of 1/500mm or better shall be used for measuring the axial deformation of test specimens.

b) LVDTs shall be set to measure the displacement of the pure bending span at positions of 15mm and 85mm from the lower surface of the test specimen as shown in Figure 2. As shown in Figure 4, the LVDTs shall be set via jigs (Figure 3) to allow their rotation when it is likely that such rotation is restricted.

5. Test procedure

a) Set the test specimen on its side with respect to its position as molded.

b) Apply the load continuously and without shock at a constant rate of $0.3 \pm 0.2\,\text{mm per minute}$ in terms of machine head speed. Before rupture occurs, record the maximum applied load from the machine indicator to three significant digits.

c) The interval of digital measurement shall be not more than 5 seconds.

d) If the fracture occurs outside of the pure bending span, discard the results of the tests.

6. Calculations

The bending moment and curvature shall be calculated using the following equations to obtain the bending moment–curvature curve.

$$M = \frac{P \cdot l}{2 \cdot 3}$$

$$\phi = \frac{\varepsilon_2 - \varepsilon_1}{d_0}$$

$M$: bending moment (N·mm)

$P$: applied load (N)

$l$: span (=300mm)

$\phi$: curvature (1/mm)

$\varepsilon_1, \varepsilon_2$: strains calculated by dividing the measured displacements of upper and lower LVDTs by contact length (100mm) (elongation is defined as positive)
$d_0$: distance between two LVDTs (=70mm)

7. Test report
The test report shall include items from the following list as required:
   a) Date of testing, test temperature
   b) Name and identification number of test specimen
   c) Age of test specimens
   d) Method of curing and curing temperature of test specimens
   e) Length of test specimen (mm)
   f) Width of test specimen (mm)
   g) Depth of test specimen (mm)
   h) Span length (mm)
   i) Mass of test specimen (kg)
   j) Apparent density of test specimen (kg/m$^3$)
   k) Maximum load (N)
   l) Maximum bending moment (N·mm)
   m) Curvature at the maximum load (1/mm)
   n) Bending moment–curvature curve
   o) Failure mode (state of cracking)
Appendix (Nonmandatory Information)
Evaluation method for tensile strength and ultimate tensile strain of Fiber-reinforced cementitious composites

Note: This appendix does not form a part of the standard but supplements matters related to the standard.

1. Scope
This appendix presents a method of calculating the tensile strength and ultimate tensile strain of fiber-reinforced cementitious composites using the maximum bending moment and curvature obtained by JCI-S-003-2007 “Method of test for bending moment–curvature curve of fiber-reinforced cementitious composites”.

2. Calculation
Calculate the tensile strength and ultimate tensile strain using the following equations to three significant digits. Express the tensile strength and ultimate strain as averages of three or more effective test results.

\[
\varepsilon_{tu,b} = \frac{\phi_u}{\phi_b} \cdot \frac{E \cdot \phi_b \cdot D \cdot x_{nl}^2}{2 \cdot (1 - x_{nl})} \\
\]

where

\[
x_{nl} = -1 + 2\cos\left(\frac{\theta}{3}\right) \quad \text{(solution of } x_{nl}^3 + 3x_{nl}^2 - 12m^* = 0 )
\]

\[
\theta = \arccos(-1 + 6m^*)
\]

\[
m^* = \frac{M_{max}}{E \cdot \phi_b \cdot B \cdot D^3}
\]

\[
x_{nl} : = x_n / D
\]
\[
x_n : \quad \text{distance from compressive edge to neutral axis (mm)}
\]
\[
D : \quad \text{depth of test specimen (=100mm)}
\]
\[
M_{max} : \quad \text{maximum moment} = \frac{P_{max}}{2 \times l / 3} \quad \text{(Nmm)}
\]
\[
P_{max} : \quad \text{maximum applied load (N)}
\]
\[
l : \quad \text{span length (=300mm)}
\]
\[
E : \quad \text{static modulus of elasticity specified by JIS A 1149 “Method of test for static modulus of elasticity of concrete” (N/mm²)}
\]
\[
\phi_u : \quad \text{curvature at the maximum load} = (\varepsilon_{tu} - \varepsilon_{tu}) / d_0 \quad \text{(1/mm)}
\]
\[
\varepsilon_{tu}, \varepsilon_{tu} : \quad \text{strains at the maximum load calculated by dividing the measured displacements of upper and lower LVDTs by contact length (100mm) (elongation is defined as positive)}
\]
\[
d_0 : \quad \text{distance between two LVDTs (=70mm)}
\]
\[
B : \quad \text{width of test specimen (=100mm)}
\]
\[
\varepsilon_{tu,b} : \quad \text{ultimate tensile strain}
\]
\[
f_{tu,b} : \quad \text{tensile strength (N/mm²)}
\]

3. Report
The test report shall include items from the following list as required:

a) Bending moment at the maximum load (Nmm)
b) Curvature at the maximum load (1/mm)
c) Tensile strength (N/mm²)
d) Ultimate tensile strain
e) Static modulus of elasticity (N/mm²)
Commentary:
Method of test for bending moment–curvature curve of fiber-reinforced cementitious composites

1. Scope
This standard mainly covers ductile fiber-reinforced cementitious composites (DFRCC)\(^1\), which shows multiple cracks under bending stress with improved ductility against bending, tensile and compressive failure. For judging the applicability of this standard, plural cracks in the pure bending span are required.

3. Test specimens
The multiple crack behavior of fiber-reinforced cementitious composites is due to the bridging effect of fibers across cracking surfaces. Placing in molds in layers or lifts should be avoided, as it causes incorrect testing values because of the lack of consistent fiber orientation. The use of a tamping rod and internal vibrator should also be avoided, as fiber orientation is affected by their use. Mold vibrators may be used.
This standard does not include specifications for curing methods. Because DFRCCs include those that develop their effective behavior only by special curing such as high-temperature steam curing, it is difficult to specify a general curing method.

Commentary:
Appendix (Nonmandatory information): Evaluation method for tensile strength and ultimate tensile strain of fiber-reinforced cementitious composites

1. Scope
Figure A1 shows typical tensile stress–tensile strain and bending moment–curvature relationships of both strain hardening-type and strain softening-type DFRCCs. Tensile strength and ultimate tensile strain evaluated by this method are as shown in the figures on the left side in comparison with actual tensile stress–tensile strain behavior. For a strain hardening-type DFRCC, the tensile strength and ultimate tensile strain evaluated by this method generally correspond to its tensile stress and strain at the maximum point obtained by uniaxial tensile test, respectively. For a strain softening-type DFRCC, the

\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
Strain hardening type & Tensile stress–tensile strain curve \hspace{1cm} Bending moment–curvature curve \\
\hline
\begin{figure}
\centering
\includegraphics[width=0.4\textwidth]{hardening_type.png}
\caption{Strain hardening type}
\end{figure}
\end{tabular}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
Strain softening type & Tensile stress–tensile strain curve \hspace{1cm} Bending moment–curvature curve \\
\hline
\begin{figure}
\centering
\includegraphics[width=0.4\textwidth]{softening_type.png}
\caption{Strain softening type}
\end{figure}
\end{tabular}
\end{table}

Figure A1 Images of tensile strength and ultimate tensile strain
tensile strength and ultimate tensile strain evaluated by this method vary depending on the extent of softening. Care should therefore be exercised, as the stress distribution assumed by this method differs from actual stress distribution in a bending stress field. Bending moment–curvature relationships obtained by this standard method for both types are as shown in the figures on the right side.

2. Calculation

The evaluation method for tensile strength and ultimate tensile strain specified by this appendix is introduced based on the following assumptions for stress distribution under the maximum bending moment.

i) The stress distribution on the compression side is triangular.

ii) The stress distribution on the tension side is uniform.

These assumptions represent a state in which the strain on the tension edge has reached the ultimate strain but the stress on the compression edge has not reached the compressive strength under the maximum bending moment as shown in Figure A2. These assumptions generally agree with actual strain and stress distributions of strain hardening-type DFRCC. On the other hand, these assumptions differ from actual distributions of strain softening-type DFRCC. For strain softening-type DFRCC, the tensile strength and ultimate tensile strain evaluated by this method correspond to certain values that show its tensile behavior as representative values. It is assumed that the elastic modulus is equal to the static modulus obtained by compression testing.

This evaluation method is introduced by the equilibrium of moments, so that the tensile stress near the tension edge has a stronger influence on the equilibrium conditions. As shown in Figure A3, the evaluated tensile strength of strain hardening-type DFRCC tends to be higher than the average tensile stress. The evaluated tensile strength of strain softening-type DFRCC presents lower values than the average tensile stress.

![Figure A2 Stress distribution assumption](image)

![Figure A3 Differences between evaluated tensile strength and average stress](image)

Compressive strain $\varepsilon_c$ at the compression edge under the maximum moment is given by the following calculation:

$$\varepsilon_c = \phi_u \cdot D \cdot x_n$$

When the compressive strain is as high as the strain at the compressive strength, it is advisable not to assume the stress distribution to be triangular. In such a case, non-linear stress distribution such as a parabolic curve may be applied. However, the position of the neutral axis is not substantially affected by
the shape of compressive stress distribution. So that the assumption of compressive stress distribution scarcely affects the evaluation of tensile strength and ultimate tensile strain.

Reference