Numerical Prediction of Crack Width in Massive Concrete Member due to Heat of Hydration

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Presented by Ingi Jang
Today’s talk will explain about three points

What is the originality point of the proposed model
- Time-dependent fracture energy
- Effect of tension stiffening
- Time-dependent RC-zone
- Linear decay in the post-cracking range

How to verify the validity of the proposed model
Comparisons between numerical analysis and experiments are performed.

Parametric study of beam structure
To investigate the influence of three parameters: reinforcement ratio, length of structure, temperature history.
How to control the thermal crack width & How to predict by numerical analysis

- For crack controlling, **the allowable maximum crack width and the stress-strength ratio** are used. Even though statistical and theoretical analysis support this idea, this index is still skeptical and not straightforward.

- When the cracking characteristics of a reinforced concrete member are evaluated by a numerical method, **the fracture characteristics of plain concrete and reinforced concrete** have been identified as the most important factors.

[Diagram showing thermal cracking and stress-strength relationship]

Architectural Institute of Japan, Recommendations for Practice of Thermal Cracking Control of Massive Concrete in Building, Japan : 2008, pp. 36-37
How to control the thermal crack width & How to predict by numerical analysis

For crack controlling, the allowable maximum crack width and the stress-strength ratio are used. Even though statistical and theoretical analysis support this idea, this index is still skeptical and not straightforward.

When the cracking characteristics of a reinforced concrete member are evaluated by a numerical method, the fracture characteristics of plain concrete and reinforced concrete have been identified as the most important factors.
How can we predict the thermal crack by numerical analysis?

- Time-dependent fracture energy and effect of tension stiffening are taken into account in FEM for predicting thermal crack width in massive concrete members.

- The subdivision of areas into a plain concrete zone and bond effect area called RC-zone has been proposed.

The fracture energy is treated as a time-dependent property.

\[ G_f(t_e) = 10 \cdot (d_{\text{max}} \cdot f_c(t_e))^{1/3} \]

where,  
- \( t_e \): temperature adjusted concrete age (days),  
- \( G_f(t_e) \): fracture energy (N/m),  
- \( d_{\text{max}} \): maximum size of aggregate (mm),  
- \( f_c(t_e) \): compressive strength of concrete at \( t_e \) (N/mm²)

\[ \sigma_c \quad \varepsilon_{\text{cr}}(t_i) \quad G_f(t_i) \]

Just after initiation of crack \( t = t_i \)

Hardening of concrete

\[ \sigma_c(t_i) \quad \Delta \varepsilon_c(t_i) \quad G_f(t_{i+1}) \]

Elapse of time after moment of crack initiation \( t = t_{i+1} \)
Time-dependent stress-strain relation of plain concrete zone

Just after initiation of crack

\[ t = t_i \]

\[
\sigma_c \quad \varepsilon_{\sigma}(t_i) \\
\sigma_c(t_i) \quad \varepsilon_{\sigma}(t_i) \\
f_i(t_i) \quad \sigma_c(t_i) \\
\sigma_c(t_{i+1}) \quad \Delta \varepsilon_c(t_i)
\]

Elapse of time after moment of crack initiation

\[ t = t_{i+1} \]

\[
\sigma_c(t_i) \quad \varepsilon_c(t_i) \\
\sigma_c(t_{i+1}) \quad \varepsilon_c(t_{i+1}) \\
\varepsilon_c(t_i) \quad \varepsilon_c(t_{i+1}) \\
\sigma_c(t_{i+1}) \quad \Delta \varepsilon_c(t_{i+1})
\]

Point where the stress and strain are zero at \( t_{i+1} \)

Point where the stress and strain are zero at \( t_i \)
Time-dependent stress-strain relation of reinforcement concrete zone

The maximum area of RC-zone depends on the tensile strength of concrete, yielding strength of rebar, and section area of rebar.

\[ A_{c,\text{max}}(t_e) = \frac{A_s \cdot f_{sy}}{f_t(t_e)} \]

Just after initiation of crack

\[ t = t_i \]

Time-dependent stress-strain relation of reinforcement concrete zone

Reinforcement concrete element

Plain concrete element

Just after initiation of crack

$t=t_i$

Elapse of time after moment of crack initiation

$t=t_{i+1}$

$h_{c,\text{max}}(t_{i+1})$

$\sigma_c(t_i)$

$\sigma_c(t_{i-1})$

$\Delta \varepsilon_c(t_i)$

$\varepsilon_{cr}(t_i)$

$\varepsilon_{cr}(t)$

$\sigma_c(t)$

$\sigma_c(t_{i+1})$

$\Delta \varepsilon_c(t_{i+1})$

Comparison of the proposed model with experiment

- The three cases of experiments which is purposed to investigate about thermal crack characteristics and influence of reinforcement in massive concrete structures were selected.
- The base structure was located underground after hardening enough. The longitudinal reinforcement ratio of base structure is 0.2%.
- The comparison with experiment is performed by focusing three points.
  - Stress history at C
  - Maximum crack width at A and B
  - Number of cracks, Spacing of cracks

Japan Concrete Institute, Guideline of massive concrete for control crack, Japan : 2008, pp. 148-174
Comparison of the proposed model with experiment

<table>
<thead>
<tr>
<th>Parameters</th>
<th>No.1-2</th>
<th>No.2</th>
<th>No.3-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of cement</td>
<td>Ordinary Portland Cement</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unit cement content</td>
<td>C = 320kg/m³</td>
<td>○</td>
<td></td>
</tr>
<tr>
<td></td>
<td>C = 380kg/m³</td>
<td>○ ○</td>
<td>○ ○</td>
</tr>
<tr>
<td>Reinforcement ratio</td>
<td>P = 0.27%</td>
<td>○ ○</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P = 0.65%</td>
<td>○</td>
<td></td>
</tr>
</tbody>
</table>

Comparison of the proposed model with experiment

Japan Concrete Institute, Guideline of massive concrete for control crack, Japan : 2008, pp. 148-174
## Comparison result of No.1-2 (C=320kg/m³, p=0.27%)

### Graphical Representation:

- **Stress of concrete (kgf/cm²)**
- **Temperature (℃)**
- **Time (days)**

### Analysis vs. Experimental Results:

- **(a) experiment**
- **(b) numerical analysis**

### Key Points:

- $\sigma_c = f_t$
- Location from the bottom of wall structure (mm)
- Maximum crack width (mm)
Comparison result of No.2 \((C=380\,\text{kg/m}^3, \, p=0.27\%)\)

- **Stress of concrete (kgf/cm²)**
  - **Time (days)**: 0, 2, 4, 6, 8, 10
  - **Temperature (℃)**: 0, 2, 4, 6, 8, 10

- **Maximum crack width (mm)**
  - **Location from the bottom of wall structure (mm)**: 0, 300, 600, 900, 1200, 1500

- **σ_c = f_t**

- **(a) experiment**
  - 3.8m, 3.8m, 2.4m, 2.8m, 2.2m

- **(b) numerical analysis**
  - Experiment, Analysis, Numerical analysis

- **Graphs**
  - Comparison of experimental and numerical analysis results for stress, temperature, and maximum crack width.
Comparison result of No.3-2 (C=380 kg/m³, p=0.65%)

- Stress of concrete (kgf/cm²)
- Time (days)
- Temperature (℃)

$\sigma_c = f_t$

(a) experiment
(b) numerical analysis

Location from the bottom of wall structure (mm)
Maximum crack width (mm)

Experiment
Numerical analysis
Parametric study of beam structure

The parametric study is performed by proposed model to investigate the influence of three parameters for the risk of cracking.

- Unit cement content / Length of structure / Reinforcement ratio

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit cement content (kg/m³)</th>
<th>Length of structure (m)</th>
<th>Reinforcement ratio (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard</td>
<td>350</td>
<td>25</td>
<td>0.405</td>
</tr>
<tr>
<td>T.1</td>
<td>250</td>
<td>25</td>
<td>0.405</td>
</tr>
<tr>
<td>T.2</td>
<td>450</td>
<td>25</td>
<td>0.405</td>
</tr>
<tr>
<td>L.1</td>
<td>350</td>
<td>10</td>
<td>0.405</td>
</tr>
<tr>
<td>L.2</td>
<td>350</td>
<td>40</td>
<td>0.405</td>
</tr>
<tr>
<td>R.1</td>
<td>350</td>
<td>25</td>
<td>0.635</td>
</tr>
<tr>
<td>R.2</td>
<td>350</td>
<td>25</td>
<td>0.850</td>
</tr>
</tbody>
</table>

Parametric study of beam structure

- The reinforcement ratio of the base structure is 0.2% in the longitudinal and it is assumed to be composed of the same concrete as the beam.
- The underlying ground is considered as large as the base structure.
Influence of unit cement content

- 450 kg/m$^3$
- 350 kg/m$^3$
- 250 kg/m$^3$

Influence of unit cement content on crack width in massive concrete member due to heat of hydration.
Influence of length of structure

- Stress of concrete (N/mm²)
- Temperature (°C)
- Crack Width (mm)

(a) 10m
(b) 25m
(c) 40m
Influence of reinforcement ratio

- **Stress of concrete (N/mm²)**
  - 0.405%
  - 0.635%
  - 0.850%

- **Temperature (°C)**
  - 350 kg/m³

- **Crack Width (mm)**
  - Maximum
  - Average

- **Time (days)**
  - 0 to 14

- **Reinforcement ratio (%)**
  - 0.405%
  - 0.635%
  - 0.850%
Conclusion

◇ In this study, the time-dependent fracture energy and the effect of the tension stiffening are taken into account in the FEM for predicting the thermal crack width.

◇ The proposed model is preliminarily validated through a comparison of the calculated results with the experimental results. Good agreement was confirmed in the case of rather higher reinforcement ratio range.

◇ The parametric study was performed to investigate the influence of the unit cement content, length of the target RC member, and the reinforcement ratio. From this parametric study, a shorter length of the target member and reducing of the maximum peak temperature seem efficient methods to control the crack width under the criterion.
Thank you very much for your kind attention
Effect of high temperature history

Based on experiments 1), it has been reported that the compressive strength of concrete, which has experienced a high temperature history in the early age, shows a slow progression in a long-term view.

Fig. 4 shows the relationship between compressive strength ratio and temperature adjusted concrete age based on experiment results 1).

The compressive strength ratio is defined by compressive strength at any temperature adjusted concrete age $t$ divided by compressive strength at 365 days in temperature adjusted concrete age.

Effect of high temperature history

In case that the maximum temperature of temperature history is higher than 45°C, the compressive strength of concrete is included in range of $\alpha$ between 0.7 and 0.9.

In this study, the coefficient of temperature history ($\alpha$) is divided in two cases. When the maximum temperature that concrete has been experienced is under 45°C, $\alpha$ is 1.0. In case of above 45°C, $\alpha$ is 0.8.

$$f_c(t_e) = \exp\left(\frac{s}{\alpha} \cdot \left[1 - \left(\frac{28}{(t_e - s_f)/T_0}\right)^{1/2}\right]\right) \cdot f_{c28} \cdot \alpha$$

Fig.4 Relationship between compressive strength ratio and temperature adjusted concrete age

The crack width is calculated from distribution of crack strain ($\varepsilon_{cr}$). The distribution of crack strain is obtained at location where the crack initiation is assumed, like A-B line. Through integration of crack strain between A and B, the crack width can be calculated.\footnote{Yasuaki Ishikawa, Hironari Ohashi, and Tadaaki Tanabe, “Proposal of calculation method for crack widths using smeared crack model”, Proceedings of the Japan Concrete Institute, Vol. 31, 2009, pp. 1555-1560}.

\begin{equation}
\varepsilon_{cr} = \varepsilon - \varepsilon_e - \varepsilon_{sh} - \varepsilon_T - \varepsilon_{creep}
\end{equation}

\[ W_{cr} = \int_{X_A}^{X_B} \varepsilon_{cr} (x) \cdot dx \]
Evaluation equation of material properties

Temperature adjusted concrete age is calculated by 1)

\[ t_e = \sum_{i=1}^{n} \Delta t_i \cdot \exp\left[13.65 - 4000/(273 + T(\Delta t_i)/T_0)\right] \]

where, \( t_e \): temperature adjusted concrete age (days), \( \Delta t_i \): number of days where a temperature \( T(°C) \) prevails, \( T_0=1°C \).

Compressive strength1) \( f_c(t_e) = \exp\left\{s \left[1 - \left(\frac{28}{(t_e-s_f)/T_0}\right)^{1/2}\right]\right\} f_{c28} \)

Tensile strength2),3) \( f_t(t_e) = 0.18 \times f_c(t_e)^{0.75} \)

Young’s modulus4) \( E_c(t_e) = 3.35 \times 10^4 \times k_1 \times k_2 \times \left(\frac{\gamma}{2.4}\right)^2 \times \left(\frac{f_c(t_e)}{60}\right)^{1/3} \)

where, \( t_0 \): 1 day, \( f_{c28} \): compressive strength of concrete at 28 days in temperature adjusted concrete age (N/mm²), \( s \): coefficient of cement type, \( s_f \): correction term for starting point of hardening, \( a \): coefficient of temperature history, \( \gamma \): air-dried mass per unit volume of concrete (t/m³), \( k_1, k_2 \): coefficients according to aggregate and admixtures.

1) CEB-FIP, Model code 1990, Bulletin’d Information, CEB, Lausanne, 1990