

Japan Concrete Institute Standard

Method of test for fracture energy of concrete by use of notched beam JCI-S-001-2003

1. Scope

This test method covers determination of fracture energy of concrete⁽¹⁾ from the load-CMOD (crack mouth opening displacement) curves of notched beams under three-point loading. Also, tension softening curves can be estimated from the load-CMOD curves obtained by this test method by following the procedure specified in the **Appendix (informative)**.

Note: ⁽¹⁾ Fiber-reinforced concrete is covered by **JCI-S-002-2003** “Method of test for load-CMOD curves of fiber-reinforced concrete using notched beams.”

2. Normative references

The following normative documents contain provisions which, through reference in this text, constitute a portion of the provisions of JCI-S-001-2003. The latest editions of these citations shall apply.

JIS A 1132: Method of making and curing concrete specimens

3. Specimens

3.1 Geometry

Specimens shall be beams of rectangular cross section with a notch at the mid-length to a depth of 0.3 times the beam depth as shown in Fig. 1.

- The depth of the cross section (D) of the specimen shall be not less than 4 times the maximum aggregate size (d_a).
- The width of the cross section (B) of the specimen shall be not less than 4 times the maximum aggregate size (d_a).
- The loading span (S) shall be $3D$. The total length of the specimen (L) shall be not less than $3.5D$.
- The notch depth (a_0) and notch width (n_0) shall be $0.3D$ and not more than 5mm, respectively.

Note: ⁽²⁾ The area above the notch subject to rupture is referred to as the ligament. The width and height of the cross section of the ligament are expressed as b and h , respectively.

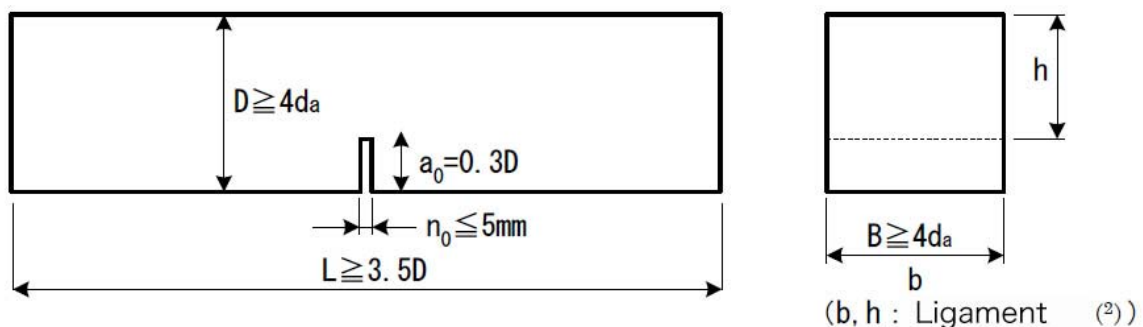


Figure 1 Specimen dimensions

3.2 Fabrication of specimens

- Specimens shall be fabricated in accordance with JIS A 1132 (Method of making and curing concrete specimens, 5. Specimens for flexure test).
- The notch shall be cut using a concrete saw when the concrete has developed sufficient strength⁽³⁾.

Note: ⁽³⁾ The notch should be cut in one side of the specimen with respect to its position as molded so that the specimen would be turned on this side for loading, with the casting surface being vertical. It is not necessary to cut the edges of the notch to specific forms, as the edge shape scarcely affects the test results. When no concrete saw is available, the notch may be formed by embedding a metal or synthetic resin plate of the specified size during placing. In this case, measures should be taken to prevent bonding between the plate and concrete.

- Specimens shall be subjected to testing in a condition immediately after completion of the specified curing procedure.
- The number of specimens shall be not less than four.
- The mass of each specimen shall be measured to the nearest 0.05 kg.

4. Apparatus

4.1 Testing machine

It is desirable to use a testing machine furnished with closed-loop control based on CMOD. Closed-loop control is desirable but not mandatory if a steady load-CMOD relationship can be measured after the peak load without rapid progress of fracture⁽⁴⁾.

Note: ⁽⁴⁾ Measurement using a testing machine that controls the displacement of the crosshead may be feasible in some cases. In such cases, it is necessary to confirm that no unstable failure occurs particularly after the peak load. When using a hand-operated testing machine, post-peak loads may be applied intermittently to avoid unstable failure.

4.2 Loading apparatus

In order to eliminate torsional action on the specimen, the loading block and one of the supports shall be rotatable around their axes in the direction coincidental with the specimen axis. Both supports shall be hinged supports having rollers. The supports shall be horizontally movable to avoid any restraint on the deformation until the specimen completely ruptures⁽⁵⁾.

Note: ⁽⁵⁾ Both supports should be movable, as the horizontal movement of the specimen is restrained at the loading block. Inserting multiple rods under both supports as shown in Fig. 2 is a simple and effective solution for a movable mechanism. In order to ensure the absence of horizontal restraint, it is advisable to press the specimen lightly by hand before applying any load to confirm smooth movement of the specimen in the horizontal direction.

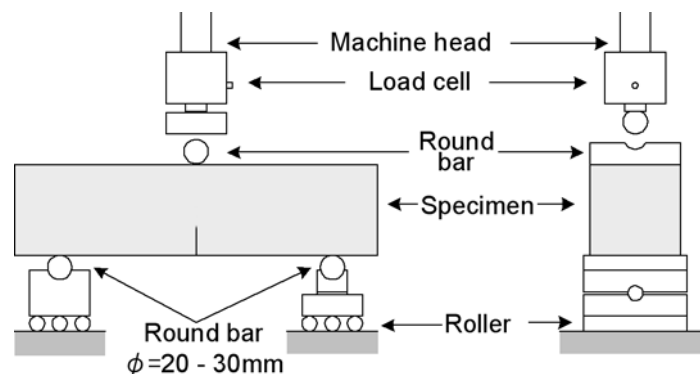


Figure 2 Loading apparatus

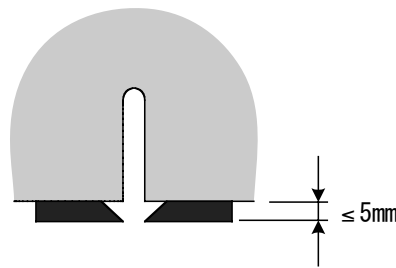


Figure 3 Knife-edges

4.3 Measuring device for loads and CMOD

The load shall be measured using a load cell with an accuracy of 1% of the estimated peak load or better. The load cell shall be fixed to the testing machine.

The CMOD shall be measured using a clip gauge that is capable of measuring to complete rupture of the specimen with an accuracy of 1/1000 mm or better. The thickness of the knife-edges to which the clip gauge is attached shall be not more than 5mm⁽⁶⁾.

Note: ⁽⁶⁾ Knife-edges may not necessarily be used when clip gauges can be directly attached to the notch. Knife-edges, which should be made of metal, should be attached as shown in Fig. 3 using an adhesive to ensure bond with the specimen. When wet testing is to be conducted with knife-edges attached to the specimen with an adhesive, the surfaces receiving the knife-edges have to be temporarily dried at the time of adhering. In this case, it is advisable that the portions other than the surfaces receiving the knife-edges be covered with a wet cloth or immersed in water to avoid drying.

5. Test procedure

- a) Set the specimen on its side with respect to its position as molded so that the notch would be located on the bottom.
- b) Load the specimen continuously and without shock. When closed loop control is executed in terms of CMOD, the preliminary load for starting loading shall be not more than 20% of the estimated peak load.
- c) The loading rate (CMOD rate) shall be 0.0005D to 0.001D/min (D = depth of specimen).
- d) Measure the load and CMOD continuously from the beginning of testing until the specimen completely ruptures. The intervals between readings by a digital measuring device shall be short enough to permit 20 or more readings before the peak load is reached.
- e) Tests are regarded as stable when the load and CMOD change slowly throughout the test without abrupt jumps. Results of tests that involve any unstable phenomenon shall be discarded.
- f) When using a manually controlled testing machine with intermittent loading after the peak load to avoid unstable fracture, adopt the envelope as the load-CMOD curve.
- g) Measure the width (b) of the broken ligament to the nearest 0.2mm at two locations and calculate the average to four significant figures.
- h) Measure the height (h) of the broken ligament to the nearest 0.2mm at two locations and calculate the average to four significant figures.
- i) Express the fracture energy and load-CMOD curve as averages of at least four specimens⁽⁷⁾.

Note: ⁽⁷⁾ To average the load-CMOD curves, calculate the averages of the loads on specimens

at the same arbitrary CMODs. The intervals between displacements for averaging should be similar to the intervals between measurements specified in section d) above.

6. Calculation

6.1 Fracture energy

Calculate the fracture energy by the following equations to three significant figures.

$$G_F = \frac{0.75W_0 + W_1}{A_{lig}}$$

$$W_1 = 0.75 \left(\frac{S}{L} m_1 + 2m_2 \right) g \cdot CMOD_c$$

where G_F = fracture energy (N/mm²)
 W_0 = area below CMOD curve up to rupture of specimen (N·mm) (4 significant figures)
 W_1 = work done by deadweight of specimen and loading jig (N·mm)
 A_{lig} = area of broken ligament (b × h) (mm²)
 m_1 = mass of specimen (kg)
 S = loading span (mm)
 L = total length of specimen (mm)
 m_2 = mass of jig not attached to testing machine but placed on specimen until rupture (kg)
 g = gravitational acceleration (9.807 m/s²)
 $CMOD_c$ = crack mouth opening displacement at the time of rupture (mm)

6.2 Tension softening curve

The tension softening curve shall be estimated in accordance with the **Appendix (informative)**.

7. Report

The test report shall include items from the following list as required:

- a) Number of specimens
- b) Curing conditions and test age
- c) Geometry of specimen
- d) Height and width of broken ligament
- e) Mass of specimen
- f) Type of testing machine
- g) Mass of loading jig
- h) Loading rate
- i) Load-CMOD curve
- j) Fracture energy
- k) Tension softening curve

Appendix (Informative)

Method of Estimating Tension softening Curve of Concrete

This **Appendix (informative)** is intended to complement matters related to the provisions of the body text but does not form a portion of the provisions.

1. Scope

This **Appendix (informative)** specifies a method of estimating a tension softening curve by poly-linear approximation using the data of load-displacement curves obtained from mode I stable failure testing on notched specimens. The tension softening curve determined by this method is a curve that expresses the relationship between the crack mouth opening displacement (CMOD) and the cohesive stress, which is employed for the analysis of mode I concrete failure by fictitious crack model.

2. Load-displacement curve

The load-displacement curve to be used for analysis should be either a load-CMOD curve determined in accordance with **JCI-S-001-2003** (Method of test for fracture energy of concrete by use of notched beams) or, in the case of fiber-reinforced concrete, a load-displacement curve determined in accordance with **JCI-S-002-2003** (Method of test for load-displacement curve of fiber reinforced concrete by use of notched beams).

3. Estimation of tension softening curve

The tension softening curve should be estimated by poly-linear approximation. Figure 1 shows the analysis flow of poly-linear approximation. Because poly-linear approximation includes numerical calculation, a program with confirmed reliability should be used for the analysis⁽¹⁾.

Note ⁽¹⁾: Programs openly available to the public with confirmed reliability include the following two, but any other program may be used provided its reliability is confirmed.

(1) Tension softening curve poly-linear approximation: a website version (LFM K method)

http://www.jci-web.jp/committee_inv0001/TSDana/fmpana.html

(2) Tension softening curve poly-linear approximation: a download version (FEM FT method)

Both programs are accessible from the website of JCI.

http://www.jci-web.jp/jci_standard/kitsutaka_dl.html

4. Estimation of elastic modulus

The elastic modulus of concrete to be used for poly-linear approximation should be determined from the secant rigidity at the point of 1/3 the maximum load on the load-displacement curve obtained from the test results, as well as the rigidity calculated on the assumption of a linear elastic body.

5. Initial cohesive stress

The initial cohesive stress to be used for poly-linear approximation should be determined from the initial slope of the load-displacement curve obtained from the test results. Carry out crack growth analyses by assuming certain cohesive stresses for early crack growth, and

determine the cohesive stress with which the analyzed and measured load-displacement relationships agree. This cohesive stress should be taken as the initial cohesive stress.

6. Calculation of load-displacement curve

The load-displacement curve should be calculated using the tension softening curve estimated by poly-linear approximation.

7. Report

The test report should include the following:

- (a) Details of the load-displacement curve test procedure
- (b) The load-displacement curve used for analysis
- (c) Outline of the analysis program
- (d) Details of analysis conditions
 - Mass of specimens
 - Element mesh of specimens or crack node arrangement
 - Support conditions for specimens
 - Tolerances for analyzed and measured load-displacement curves by poly-linear approximation
 - Other
- (e) Elastic modulus
- (f) Tension softening curve
- (g) Comparison between calculated and tested load-displacement curves

Commentary

Method of Estimating Tension softening Curve of Concrete

1. Scope

The fracture phenomenon of concrete is strongly related to the behavior of the fracture process zone (FPZ) present at the tip of a completely open crack. Because of the softening phenomenon occurring in such a FPZ, the fracture phenomenon of concrete shows nonlinear features. The fictitious crack model[1] shown in Fig. 1 was proposed to model such nonlinear fracture behavior. In this model, a FPZ of concrete is represented by a mechanical model in which cohesive stresses act on a single fictitious crack. The cohesive stresses are changed according to the changes in the width of the fictitious crack (opening displacement). The softening phenomenon can be numerically expressed by reducing the cohesive stress as the fictitious crack width increases. The relationship between the cohesive stress and the fictitious crack width (opening displacement) in this process is the tension softening curve (Fig. 2).

The present estimation method requires data for a load-displacement curve obtained from mode I stable failure testing using notched specimens. The test methods include three-point bending testing using notched beam specimens, wedge insertion cleavage testing, compact tension testing, and direct tension testing, etc.

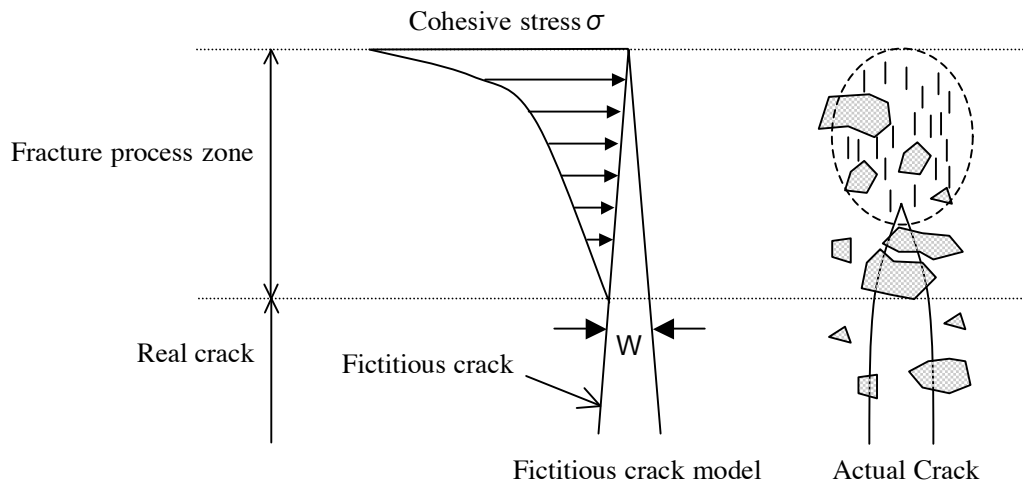


Fig. 1 Fictitious crack

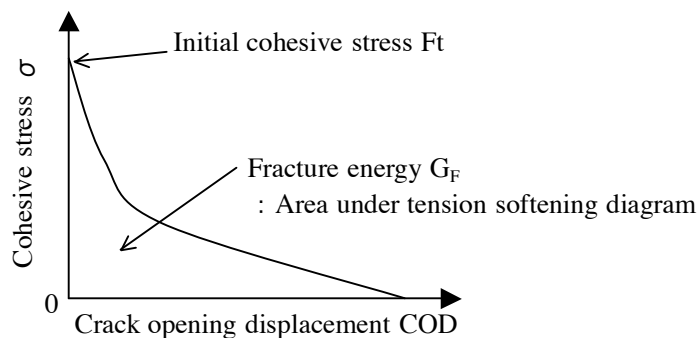


Fig. 2 Tension softening diagram

2. Poly-linear approximation

2.1 Outline

Estimation of a tension softening curve by poly-linear approximation[2-4] requires repeated crack growth analyses. Crack growth analysis by the present estimation method is based on a fictitious crack model. The specimen is modeled into a fictitious crack area representing the fracture zone and the other area representing a linear elastic body. Failure is assumed to occur only in the fictitious crack area. The load as an external force and the cohesive stress acting on the fictitious crack surfaces at a given length of the crack can be calculated by assuming the equilibrium conditions at the tip of the crack to be the tensile strength (hereafter referred to as initial cohesive stress) and stress intensity factor and setting the compatibility conditions of the fictitious crack area[5, 6]. Particularly in crack growth analysis used for poly-linear approximation, the load-displacement relationship can be calculated by repeatedly solving simultaneous linear equations, in which the unknown is the CMOD, based on a tension softening curve partly approximated to straight lines as a constitutive law[7].

Poly-linear approximation is a method of estimating a tension softening curve by back analysis based on a load-displacement curve derived from test data. In crack growth analysis using a fictitious crack model, the slope of the tip of the tension softening curve to be used as a constitutive law is changed, so as to determine the optimum slope, with which the measured and analyzed load-displacement relationships agree. This process is sequentially repeated to determine the overall shape of the tension softening curve. During this procedure, the tension softening curve that has already been determined is used as a constitutive law for subsequent calculation steps. Figure 3 shows the relationship between the load-displacement curve and the poly-linearly approximated softening curve obtained by back analysis.

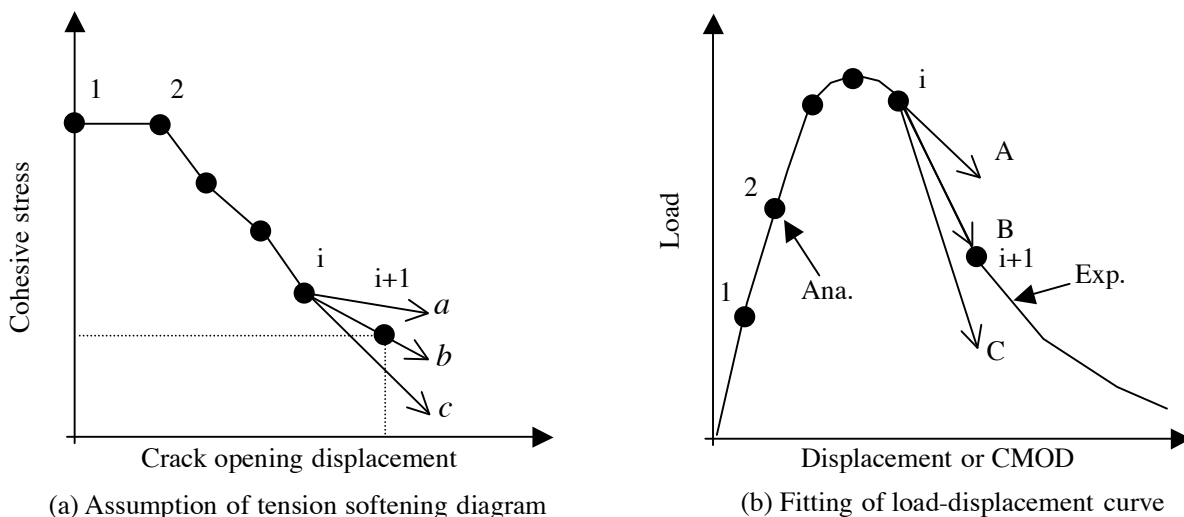


Fig. 3 Poly-linear inverse analysis

Specifically, lines *a*, *b*, and *c*, for instance, are assumed as tension softening curves starting from the *i*th point as shown in Fig. 3(a), and a load-displacement curve resulting from the crack growth to the next node is analyzed for each of the three lines. Points A, B, and C are thus determined by analysis as the next points corresponding to lines *a*, *b*, and *c*, respectively (Fig. 3(b)). If the measured load-displacement curve agrees with point B, then a point on line

b at a distance equal to the CMOD at point B is determined as the $(i+1)$ th point of the tension softening curve.

A smooth load-displacement relationship is suitable for the present estimation method, particularly in the early stage. The tension softening curve may not be estimated unless the load-displacement curve is smooth. When the measurement data oscillate, it is advisable to process the data to obtain a smooth curve.

2.2 Analysis programs currently available to the public

Two programs for poly-linear approximation have been made available to the public. These are characterized as follows:

Estimation of a tension softening curve by poly-linear approximation requires repeated crack growth analyses. To configure crack equations (equilibrium and compatibility condition equations for a fictitious crack)[5], the analysis program formulated by Uchida (hereafter referred to as the FEM FT method http://www.jci-web.jp/jci_standard/kitsutaka_dl.html) employs finite elements and adopts the tensile stress at the tip of the fictitious crack as the equilibrium condition of the crack equation. The FEM FT method, which is written in the FORTRAN programming language, consists of the five component programs given below and is made available to the public. Figure 4 shows the work flow of estimating a tension softening curve using the following programs.

ini.for:	program for determining the coefficients of the crack equation
ym.for:	program for determining the elastic modulus
ft.for:	program for determining the initial cohesive stress
soft.for:	program for determining the overall tension softening curve
pd.for:	program for determining the load-displacement relationship using a given tension softening curve

On the other hand, the analysis program configured by Kitsutaka (hereafter referred to as the LFM K method) employs a functional solution of linear elastic fracture mechanics (LFM) for formulating crack equations[2]. The stress intensity factor, K , is adopted as the equilibrium condition at the tip of the fictitious crack, and the shape function of the specimen is determined from the results of numerical analysis by LFM. The analysis program currently made available to the public for estimating a tension softening curve is for use on the Web (http://www.jci-web.jp/committee_inv0001/TSDana/fmpana.html). It requires the following conditions:

Specimen size:	100 by 100 by 400 mm
Span length:	300 mm
Notch depth:	30 mm (midspan)
Three-point loading testing	
Load-CMOD curve data	

The tension softening curves of normal concrete with a W/C of 0.65 and steel fiber-reinforced concrete with a W/C of 0.40 and fiber content of 1.0% were estimated using these two back analysis programs to investigate the effect of the method on the estimated curve. As a result, the shapes of tension softening curves estimated by these different methods nearly agreed as shown in Fig. 5, being scarcely affected by the difference in the analysis methods. Both programs are therefore proven viable, leading to the same results.

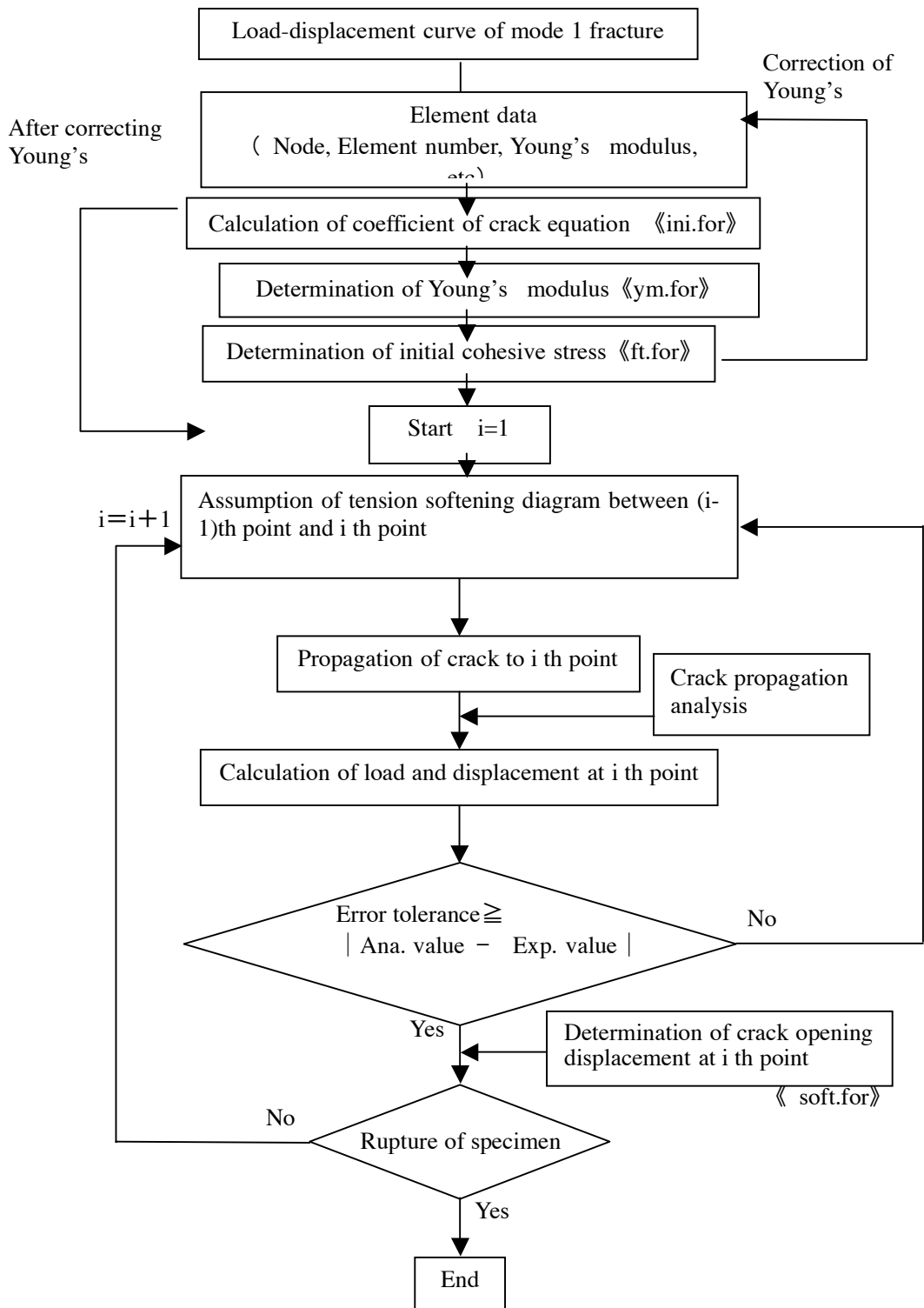


Fig. 4 An example of flow chart of poly-linear inverse analysis method

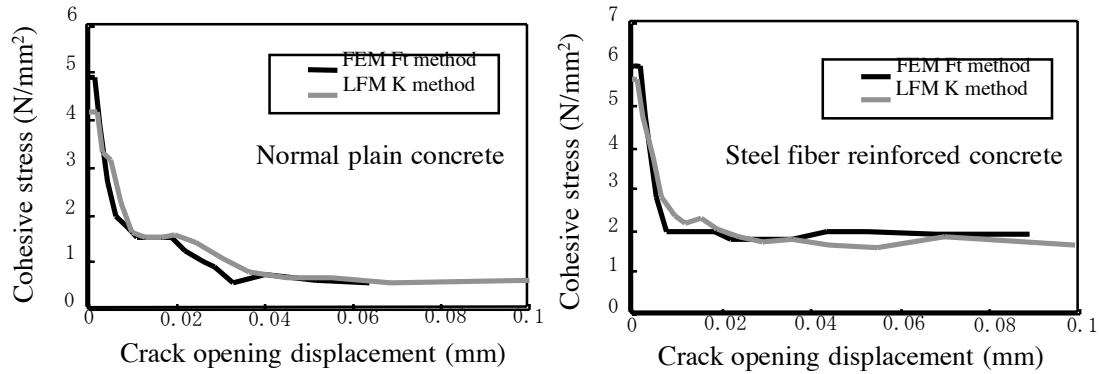


Fig. 5 Tension softening diagrams determined through two different analysis

3. Estimation of elastic modulus

It was decided that the elastic modulus necessary for back analysis be estimated from the secant rigidity at a loading point 1/3 the maximum load on the load-displacement curve based on test data.

Numerical analysis should be carried out on the assumption that the specimen is a linear elastic body. It is only necessary to estimate the elastic modulus with which the calculation results agree with the secant rigidity at a point 1/3 the maximum load on the load-displacement curve obtained from test results. It is advisable that the same numerical analysis method as the one for crack growth analysis be used for minimizing errors.

Specifically, the elastic modulus can be estimated as follows: Determine the load 1/3 the maximum load, P_0 , and the displacement under that load, δ_0 , on the load-displacement curve based on the test data. Then analyze the load-displacement relationship by assuming an appropriate elastic modulus, E_{c1} , to determine the analyzed displacement corresponding to P_0 , δ_c . Determine the elastic modulus, E_{c2} , so that δ_0 would agree with δ_c . The elastic modulus of concrete, E_{c2} , can be estimated by Eq. (1).

$$E_{c2} = E_{c1} \times \frac{\delta_c}{\delta_0} \quad (1)$$

4. Initial cohesive stress

When estimating a tension softening curve by poly-linear approximation or any other methods basically using back analysis, the initial cohesive stress cannot be uniquely determined. For this reason, it was decided that the initial cohesive stress be determined as follows: Assume certain cohesive stresses for initial crack growth (tension softening curves of a completely plastic type), carry out crack growth analysis, and determine the cohesive stress with which the measured and analyzed load-displacement relationships agree, to be taken as the initial cohesive stress.

There have currently been two specific methods of estimating the initial cohesive stress: one that determines the initial cohesive stress from the agreement of load-displacement relationships at a point of time at which the crack has grown to a given length, e.g., 5 mm, and the other that determines it from the agreement of early load-displacement curves. In the former method, crack growth analysis is carried out using tension softening curves of a

completely plastic type with varying cohesive stresses. When the analyzed load-displacement relationship at the point at which the crack has grown to a length of, e.g., 5 mm agrees with the measured relationship within the specified tolerances, the cohesive stress is taken as the initial cohesive stress. This is concerned with the load-displacement relationship when the crack growth length reaches 5 mm, while disregarding the earlier behavior of the crack. In the latter method, crack growth analysis is carried out using tension softening curves of a completely plastic type with varying cohesive stresses. When the analyzed load-displacement relationship from the beginning agrees with the measured relationship within the specified tolerances, the cohesive stress is taken as the initial cohesive stress. In contrast to the former method, the load-displacement relationships from the beginning are required to agree within the specified tolerances.

To judge the agreement of the analyzed and measured load-displacement relationships, it is advisable to check the difference between the measured and analyzed values at the same displacement. In this case, the tolerances may be specified in either an absolute value (e.g., 1 N) or a relative value (e.g. 1%) from the measured values.

In the latter case, a number of initial cohesive stresses may be estimated depending on the specified tolerances. In such a case, it is advisable to select the cohesive stress leading to the longest crack length.

It has been confirmed that different initial cohesive stress values lead to no marked differences in the shapes of tension softening curves[8, 9].

5. Report

Reports should be made regarding the listed items. When the load-displacement curve of item (2) has been smoothed, the smoothing method should be described. When a program available to the public is used in regard to item (3), the “name of the program” and the “reference (URL, etc.)” should be reported. When a new analysis program has been formulated, the report should include the outline of the method, such as the “method of estimating the initial cohesive stress,” “equilibrium condition at the tip of the crack,” and “method of calculating the shape functions.”

<References>

- [1] Hillerborg, A., Modeer, M. and Petersson, P.E.: Analysis of Crack Formation and growth in Concrete by Means of Fracture Mechanics and Finite Elements, Cement and Concrete Research, 6, pp.773-782, 1976
- [2] Kitsutaka, Y. : Fracture Parameters by Poly-linear Tension Softening Analysis, Journal of Engineering Mechanics, ASCE, Vol.123, No.5, pp.444-450, 1997
- [3] Kurihara, N., Ando, T., Kunieda, M. Uchida, Y. and Rokugo, K. : Determination of Tension Softening Diagram of Concrete by Poly-linear Approximation Analysis and Failure Behavior of Fiber Reinforced Concrete, Proc. Japan Soc. Civil. Eng., 532/V-30, 119-129, 1996.2.s
- [4] Nanakorn, P. and Horii, H.: Back Analysis of Tension-Softening Relationship of Concrete, Proc. Japan Soc. Civil. Eng., 544/V-32, pp.265-275, 1996.8.
- [5] JCI : " Colloquium for Fracture Mechanics of Concrete Structures", Japan Concrete Institute, 1990
- [6] JCI : "Applications of Fracture Mechanics to Concrete Structures", Japan Concrete Institute, 1993
- [7] Modeer M.: A fracture mechanics approach to failure analyses of concrete materials, Div. of Building Mat., Univ. of Lund, Report TVBM-1001,1979
- [8] JSCE : "Size Effect and Tension Softening Diagram of Concrete", Concrete Library 18, pp.6-15, 1997
- [9] Uchida, Y. and Barr, B.I.G.: Tension Softening Curves of Concrete Determined from Different Test Specimen Geometries, Fracture Mechanics of Concrete Structures (edited by Mihashi, H. and Rokugo, K.), pp.387-398, 1998

Report of the technical committee on the test method for fracture property of concrete (JCI-TC992)(2001.5)

Member of technical committee on the test method for fracture property of concrete (JCI-TC992)(1999.4-2001.3)

Chairman : Yoshinori Kitsutaka
Co-chair : Yuichi Uchida
Co-chair : Yoshio Kaneko
 : Hiroshi Akita
 : Satoshi Ishiguro
 : Seiichiro Ishihara
 : Hiroshi Ohtsuka
 : Tokunao Ohoka
 : Tetsushi Kanda
 : Minoru Kunieda
 : Tetsuhiko Kurihara
 : Naoyuki Koshiishi
 : Yukihiro Sato
 : Yasuji Shinohara
 : Nobuaki Shirai
 : Shigeharu Nakamura
 : Hiroshi Fukuyama
 : Toshiaki Hasegawa
 : Maki Matsuo
 : Shoji Matsuo
 : Takashi Matsumoto
 : Hoirozo Mihashi
Communicated member
 : Masayasu Ohtsu
 : Hideaki Horii
 : Kiyoshi Murakami