Shear stiffness degradation of RC walls under varying axial loads

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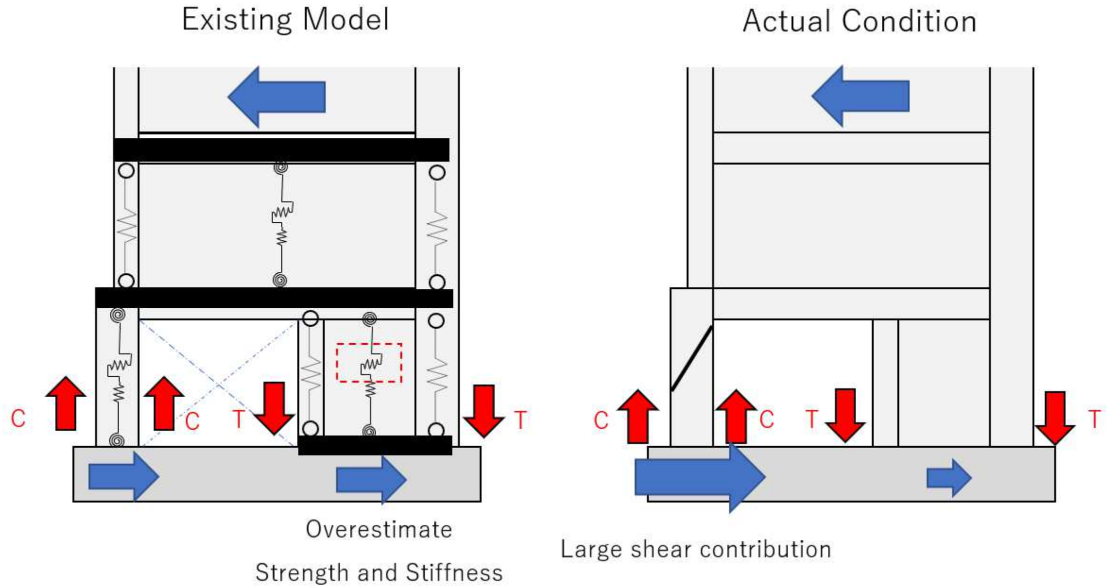
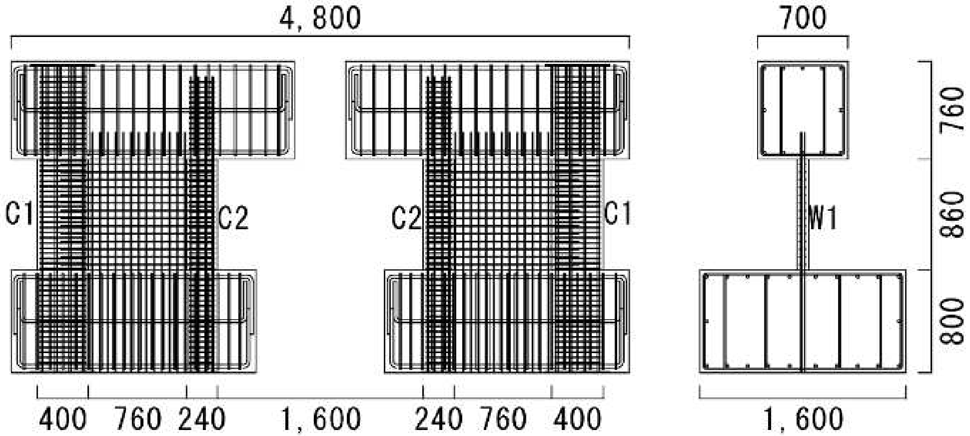
Background



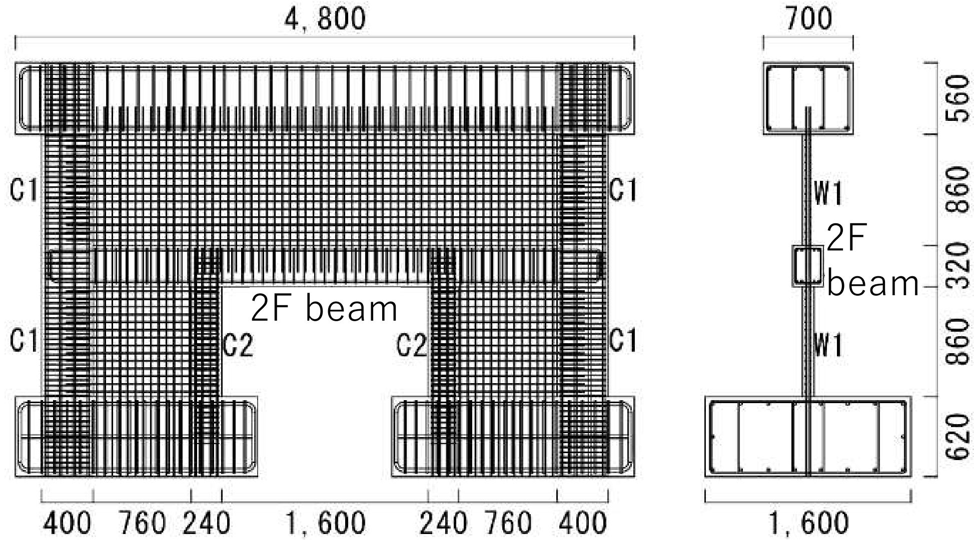
- The degrading factor of shear stiffness did not consider the effect of the varying axial load in Japanese design model.
- The shear stiffness of the tensile wall did not decrease, the contribution of the shear force from the column was underestimated as above.
- A modified frame analysis method is proposed using FEM algorithm, and its accuracy is verified by loading test of RC walls under varying axial load

Specimen

Wall	90mm (p_w=1.17%)	Boundary column	C1: 400×480mm(p_g=1.66%) C2: 240×240mm(p_g=1.49%)

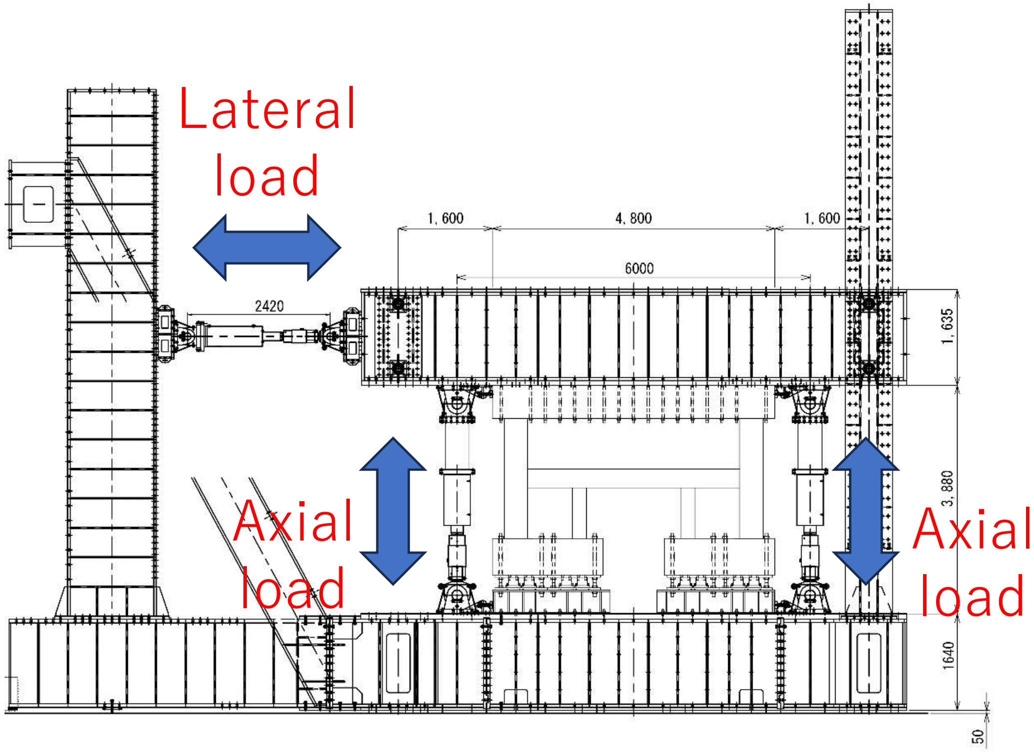


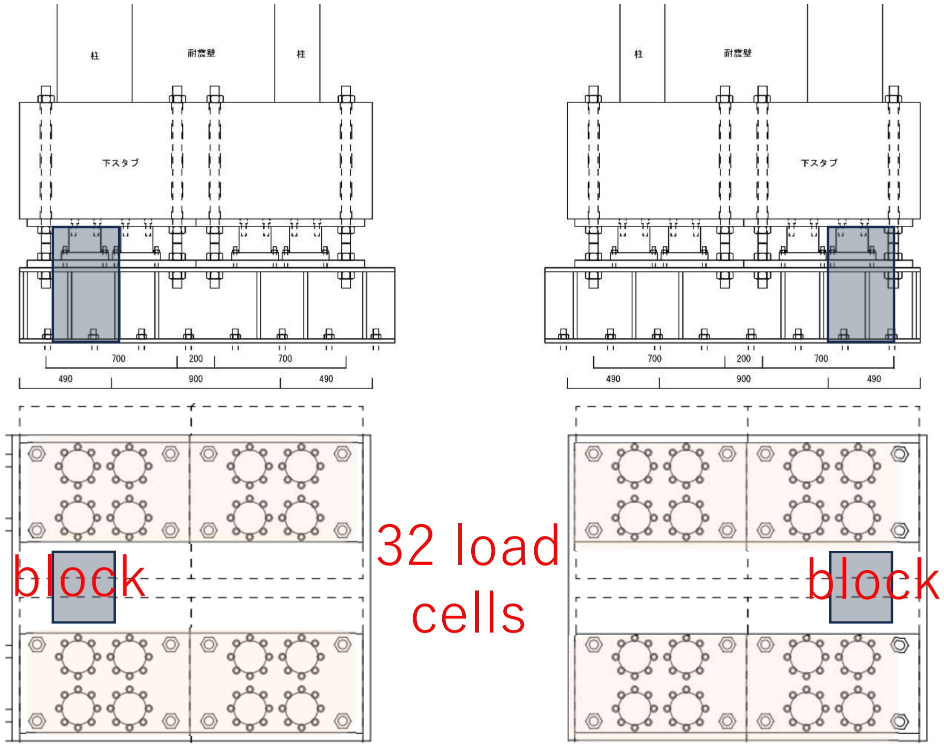
Single-story specimen	
Axial load ratio	0.08
M/Q	8m



two-story specimen	
Axial load ratio	0.06
M/Q	6m

Loading System





Test Result（Single-Story）M/Q=8m, Axial load ratio 0.08

The strength does not deteriorate up to 2% drift ratio

Failure mode was flexural failure

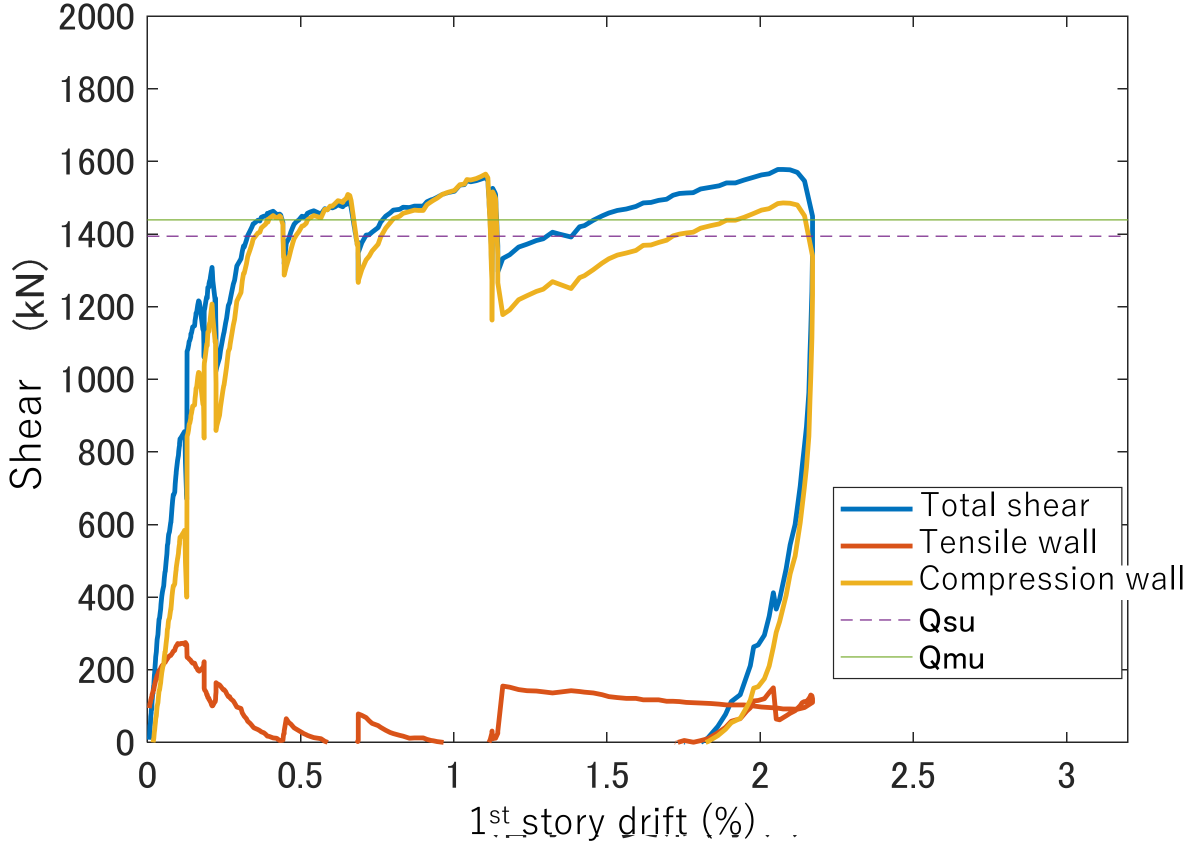
(axial compression failure on the compression side, horizontal shear cracks on the tension side)

Envelop Curve (Single-story specimen)

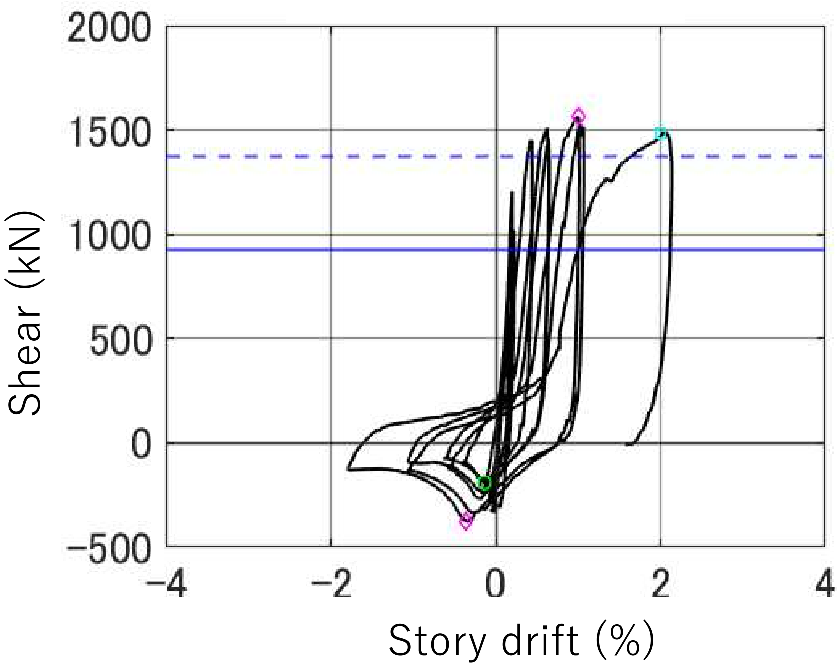
The compression wall contributes almost all of shear force

the tension side falls to approximately 0 as deformation increases from 200kN.

Total shear corresponds to the calculation flexural strength as a single wall with an opening.



Load-displacement relation of single wall



Maximum shear force of compression side wall

$Q_w = 1565\ kN$ $t_u = 16.3$(MPa)

$\tau_u/\sigma_B = 0.31$ →Flexural failure

Design strength of one-side compressive wall

with the M/Q is 1.4m

$Q_{wsu} = 927\ kN$

→ Under estimate walls shear strength with high axial forces (about 0.5 times)

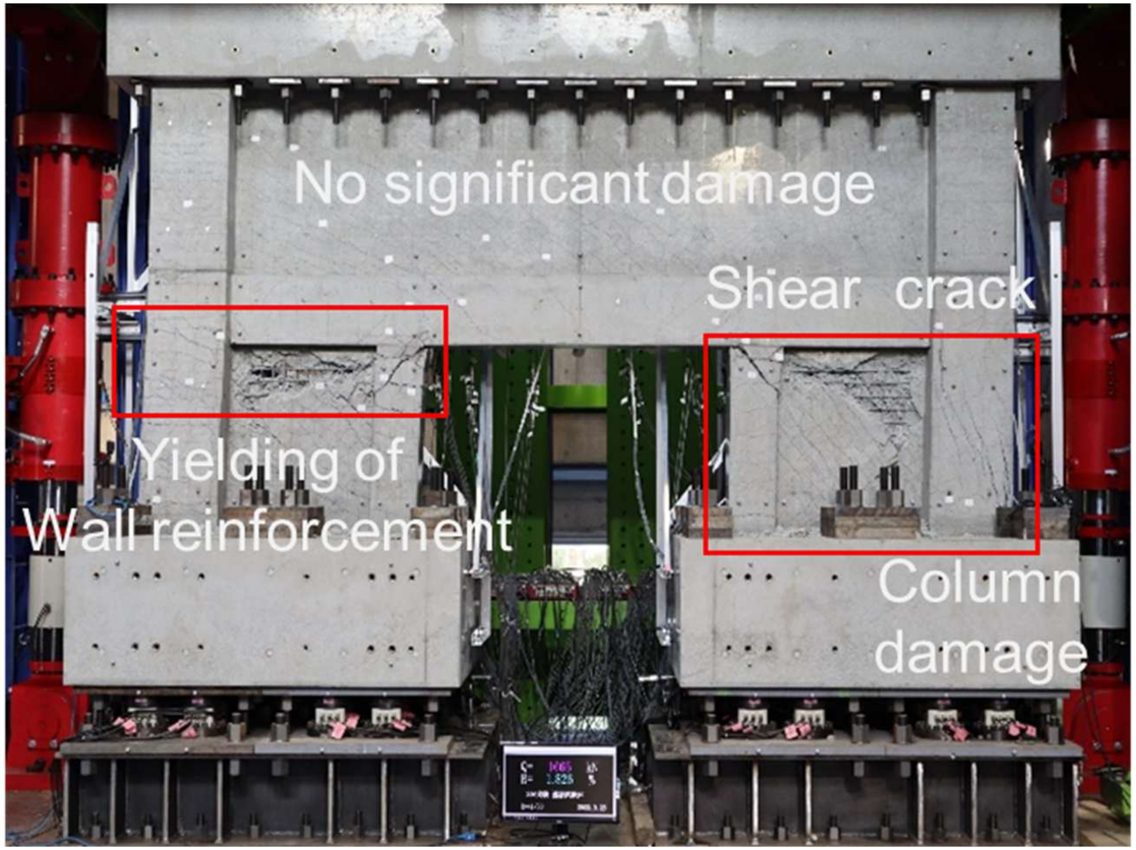
$Q_{wsu} = 1379\ kN$

(without upper limit of equivalent wall thickness ratio t_e <1.5t)

Test Result（Two-Story）M/Q=6m, Axial load ratio 0.06

Transverse reinforcement yields at 0.25% drift, and strength deteriorated gradually.

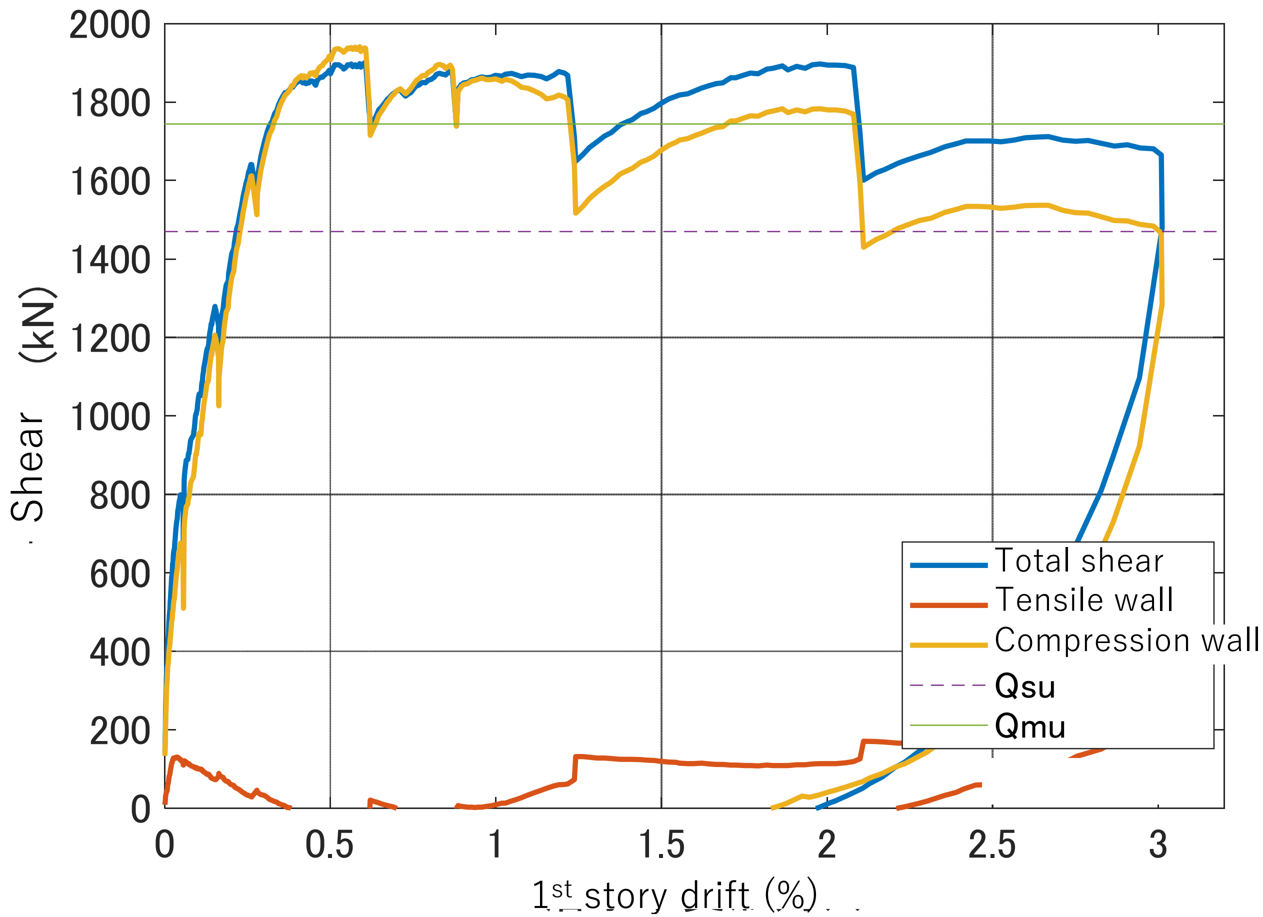
The shear compression failure occurs on the compression side and horizontal shear cracks on the tension side.



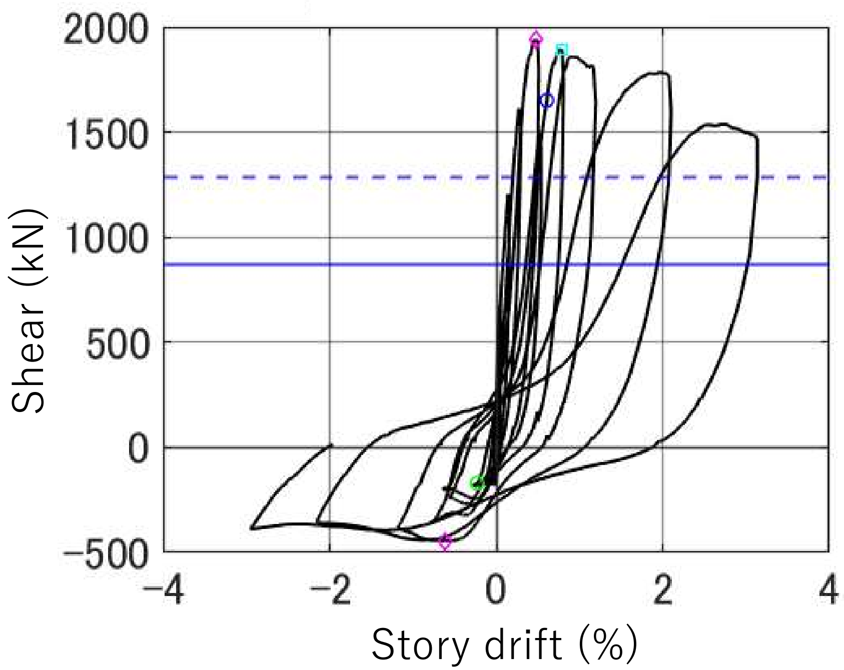
Envelop Curve (Two story specimen)

The maximum shear force far exceeds the design shear strength of a single opening wall.

The compression wall contributes almost all of shear force



Load-displacement relation of single wall



Maximum shear force of compression side wall

$Q_w = 1941\ kN$ $t_u = 20.0$(MPa)

$\tau_u/\sigma_B = 0.40$ → shear degradation

Design strength of one-side compressive wall

with the M/Q is 1.4m

$Q_{wsu} = 879\ kN$

→ Under estimate walls shear strength with high axial forces (about 0.5 times)

$Q_{wsu} = 1299\ kN$

(without upper limit of equivalent wall thickness ratio t_e <1.5t)

Japanese design shear strength（Hirosawa Formula）

Empirical Formula derived by database of wall tests

Limited test data in axial load, shear span ratio, and material strength

$$Q_{wsu} = \left\{ \frac{0.068 p_{te}{}^{0.23} (F_c + 18)}{(M/QD + 0.12)} + 0.85\sqrt{p_{wh}\sigma_{wy}} + 0.1\sigma_0 \right\} t_e j$$

p_{te} : Equivalent Tensile Reinforcement ratio (%)

p_{wh} : Transverse reinforcement ratio (<0.012)

σ_{wy} : Transverse reinforcement yielding strength

M/QD : Shear span ratio ($1 \leq M/QD \leq 3$)

t_e : equivalent thickness equalized to square section (<1.5t)

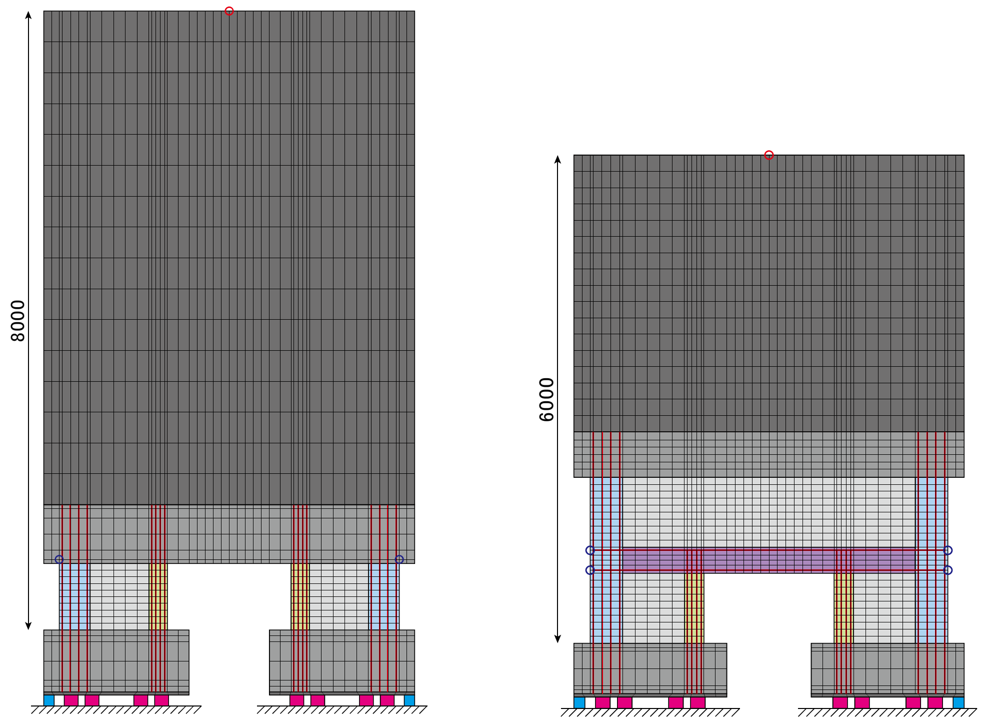
σ_0 : Axial stress on full section

FEM analysis

Pushover analysis was performed using two-dimensional finite elements.

The superstructure was treated as a rigid body with external force heights of 8m and 6m.

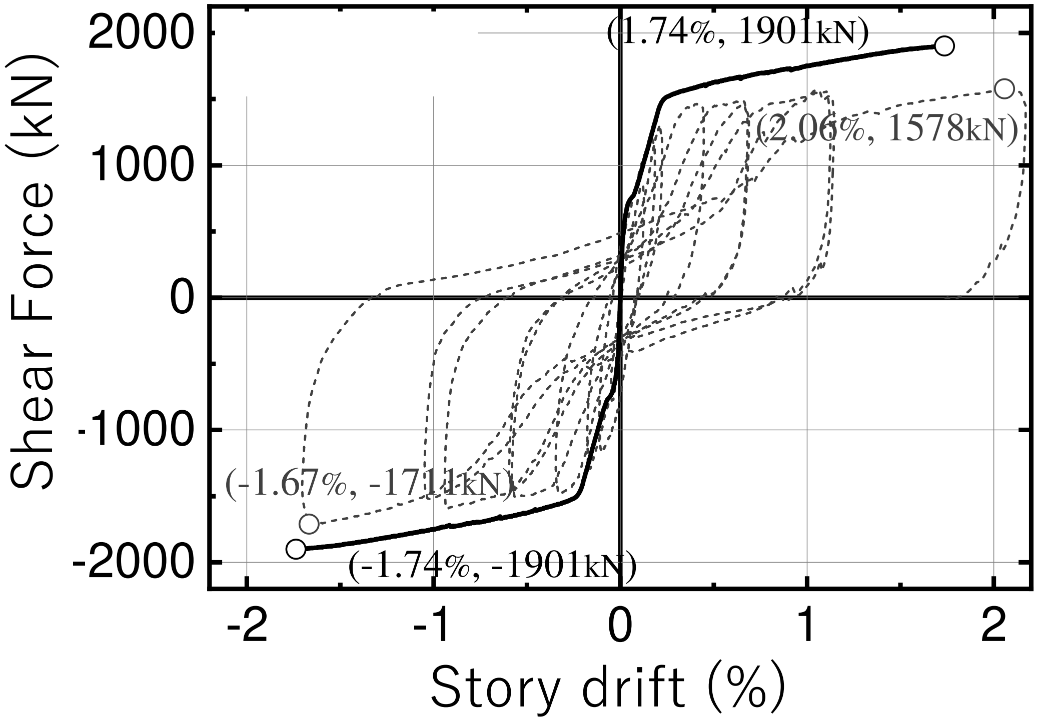
Concrete: Modified Ahmad model, embedded steel bars: Modified Menegotto-Pinto model (truss)

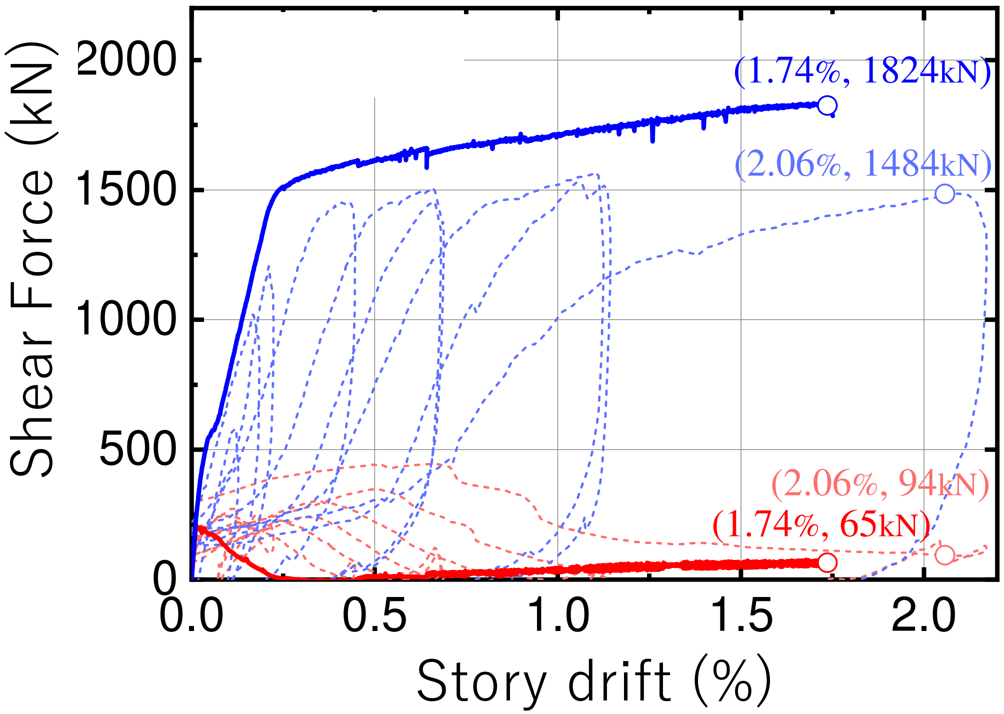


FEM analysis on single-story test

The analysis results slightly overestimated the strength, but shows similar envelop curve.

Compression side wall contributes almost all of the shear force as well as test.

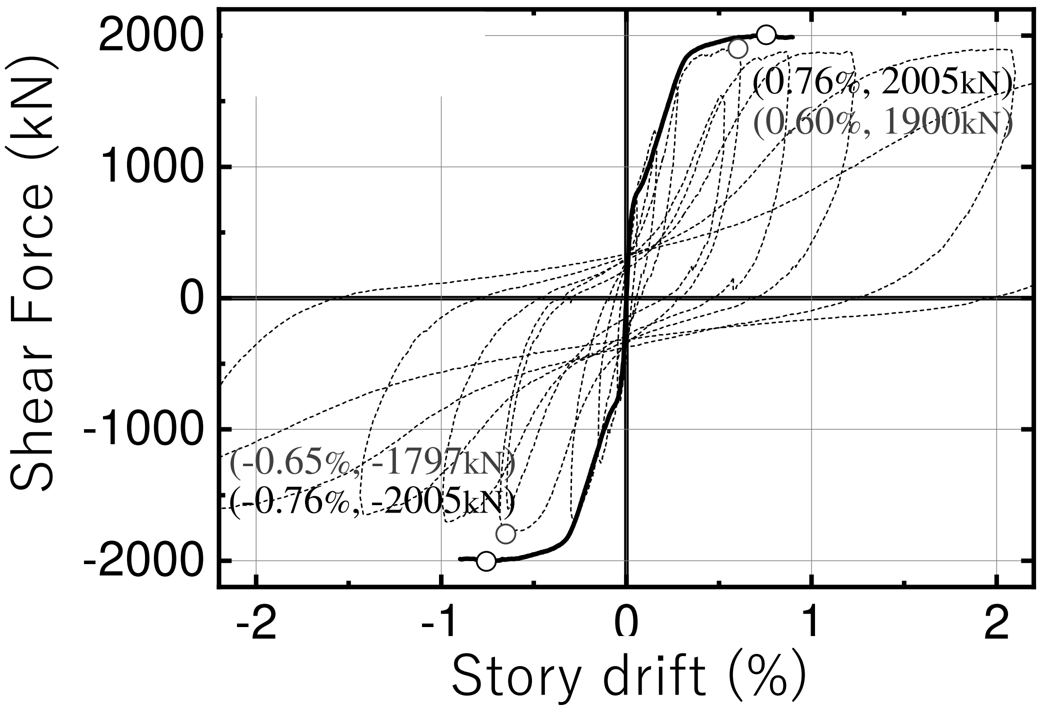


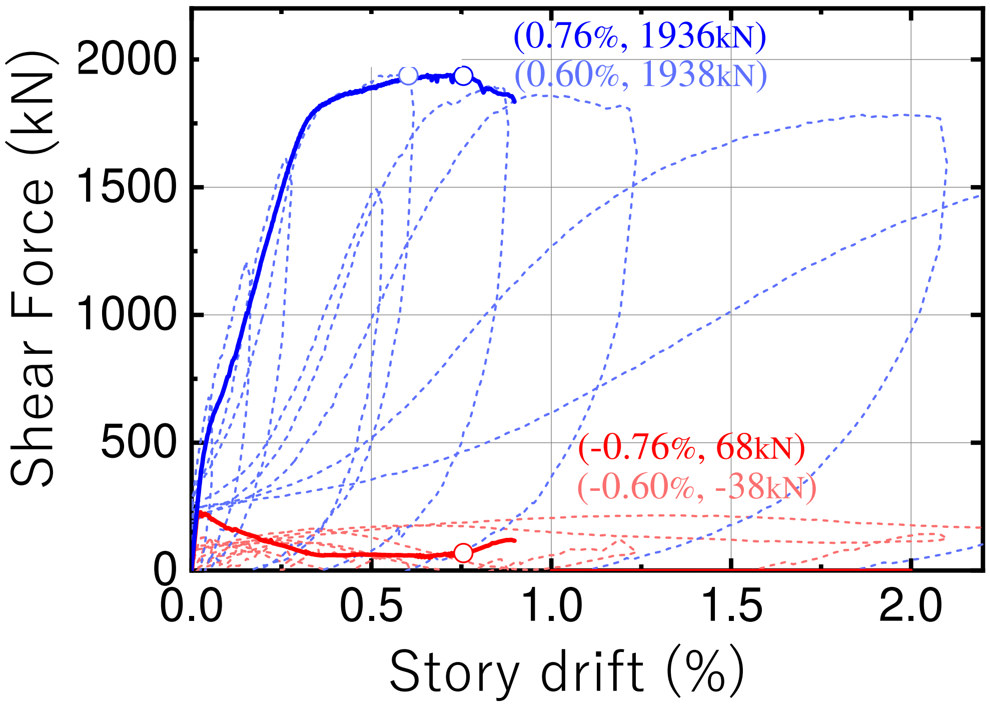


FEM analysis on two-story test

The envelope curves are consistent with the test results.

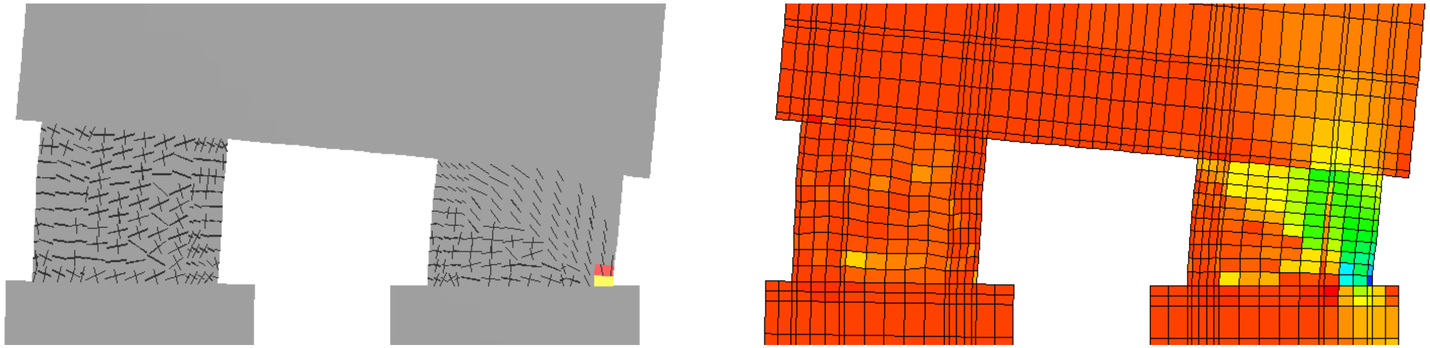
Compression side wall contributes almost all of the shear force as well as test.



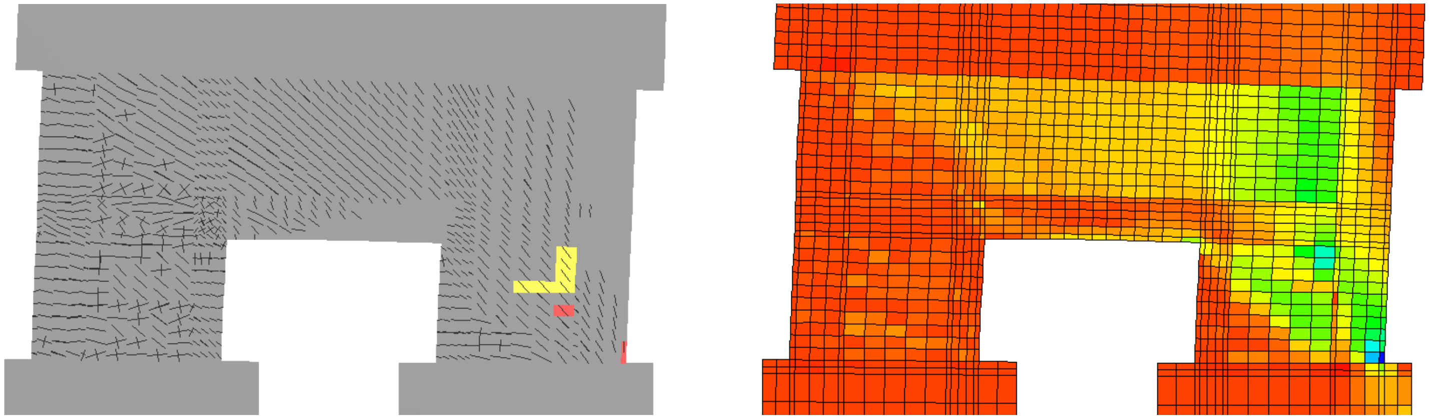


Stress and Damage Pattern on FEM

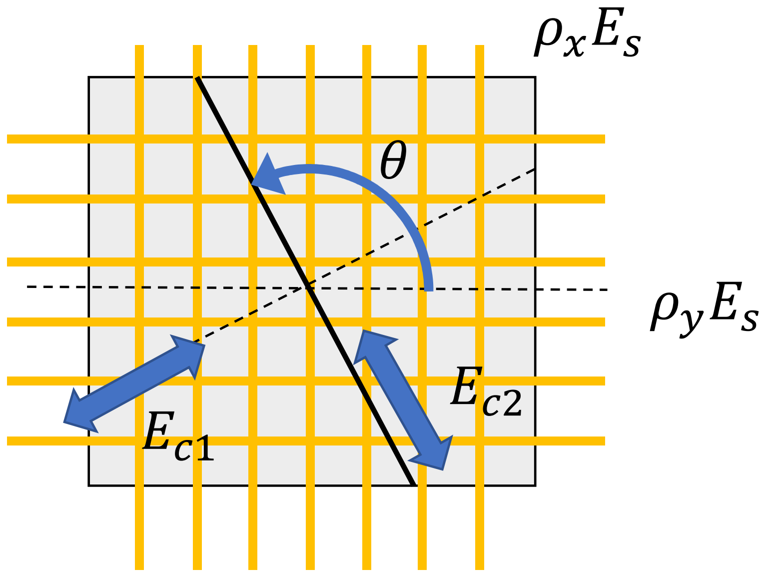
no axial compressive failure occurred in the wall panels, and only the outer column bases softened.



the outer column base and wall panel softened, and the compression strut shifted toward the inner column.



Stress- Strain relation in gaussian point（FEM）



Concrete　　　　Steel rebar

$$\begin{pmatrix}\sigma_x\\ \sigma_y\\ \tau\end{pmatrix} = \left[[T] \begin{bmatrix} E_{c1} & 0 & 0 \\ 0 & E_{c2} & 0 \\ 0 & 0 & G_c \end{bmatrix} [T]^t + \begin{bmatrix} \rho_x E_s & 0 & 0 \\ 0 & \rho_y E_s & 0 \\ 0 & 0 & 0 \end{bmatrix} \right] \begin{pmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma\end{pmatrix}$$

$$[T] = \begin{bmatrix} cos^2\theta & sin^2\theta & cos\theta sin\theta \\ sin^2\theta & cos^2\theta & -cos\theta sin\theta \\ -2cos\theta sin\theta & 2cos\theta sin\theta & (cos^2\theta - sin^2\theta) \end{bmatrix}$$

Calculating the invert matrix component

Shear stiffness $K_s = \frac{\tau}{\gamma} = \frac{E_{c1}E_{c2}}{2(E_{c1}sin^2\theta + E_{c2}cos^2\theta)}$

Principal stress direction$\theta$and concrete axial stiffness function

Proposal of "Axial-Shear model"

Invert Matrix

$$\begin{pmatrix}\sigma_x\\ \sigma_y\\ \tau\end{pmatrix}=\begin{bmatrix}\bigcirc & \bigcirc & \bigcirc\\ \bigcirc & \bigcirc & \bigcirc\\ K_1 & K_2 & K_3\end{bmatrix}\begin{pmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma\end{pmatrix} \Rightarrow \begin{pmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma\end{pmatrix}=\begin{bmatrix}\bigcirc & \bigcirc & \bigcirc\\ \bigcirc & \bigcirc & \bigcirc\\ F_1 & F_2 & F_3\end{bmatrix}\begin{pmatrix}\sigma_x\\ \sigma_y\\ \tau\end{pmatrix}$$

Lateral stress is 0

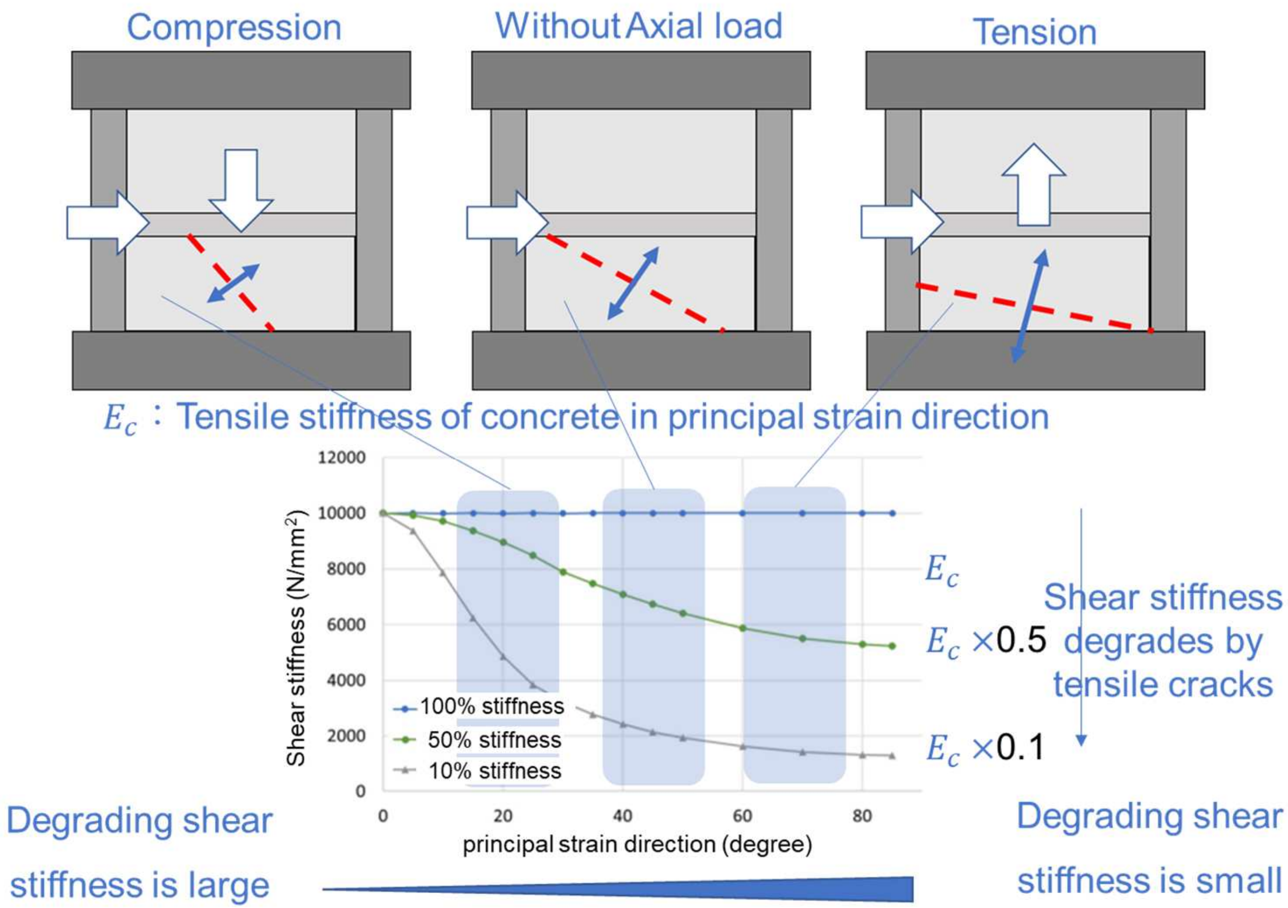
Shear stiffness $K_s=\frac{\tau}{\gamma}=\frac{\tau}{F_1\sigma_x+F_3\tau}=\frac{1}{\left(2F_1\frac{(\sigma_x/2)}{\tau}+F_3\right)}=\frac{1}{\left(2F_1\frac{1}{tan2\theta}+F_3\right)}$

Substitute F_1, F_3 and Transformation

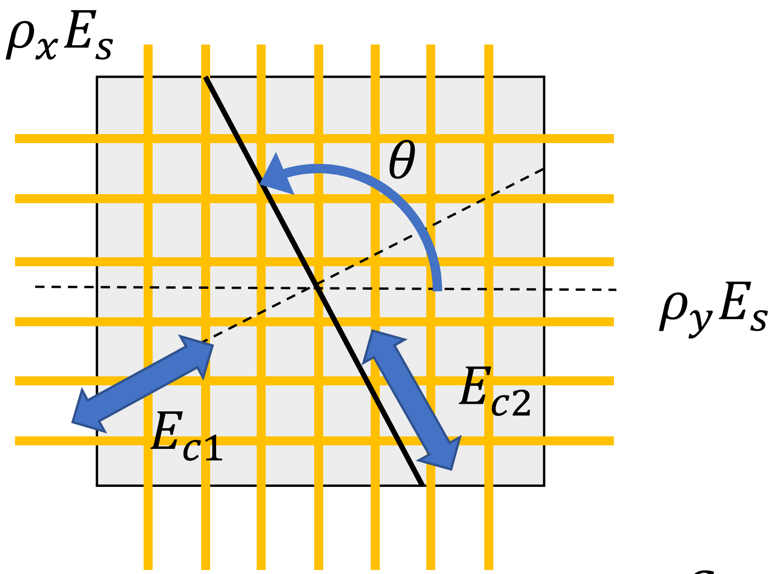
$$K_s=\frac{E_{c1}E_{c2}}{2(E_{c1}sin^2\theta+E_{c2}cos^2\theta)}$$

(Function of Principal stress direction θ
and Concrete axial stiffness E_c in those directions)

Degrading shear stiffness – principal strain direction



Shear stiffness from material stiffness in principal direction



Concrete | Steel rebar

$$\begin{pmatrix}\sigma_x\\ \sigma_y\\ \tau\end{pmatrix} = \left[[T]\begin{bmatrix}E_{c1} & 0 & 0\\ 0 & E_{c2} & 0\\ 0 & 0 & G_c\end{bmatrix}[T]^t + \begin{bmatrix}\rho_x E_s & 0 & 0\\ 0 & \rho_y E_s & 0\\ 0 & 0 & 0\end{bmatrix}\right]\begin{pmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma\end{pmatrix}$$

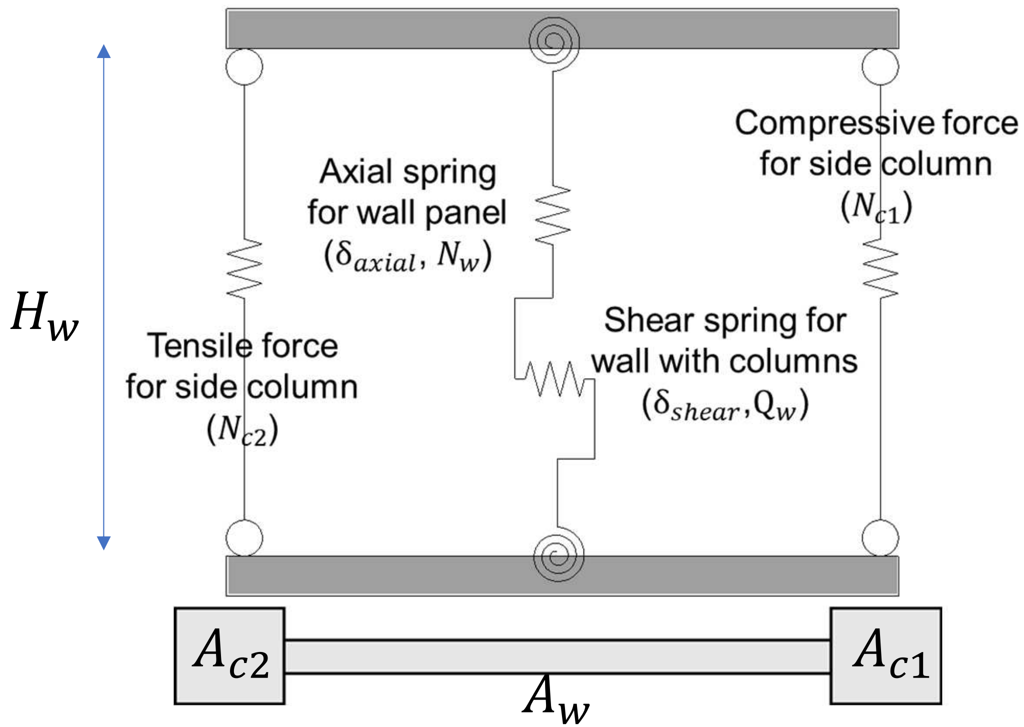
Reflecting the rebar term in the formula

$$G = \frac{(c + \rho_y E_{sy} b)c}{2ac + \rho_y E_{sy}(c + ab)}$$

$$a = E_{c1} sin^2\theta + E_{c2} cos^2\theta, \qquad b = E_{c1} cos^2\theta + E_{c2} sin^2\theta, \qquad c = E_{c1}E_{c2}$$

E_{c1}, E_{c2}: Elastic modulus of concrete in the tensile and compressive directions
θ: principal direction, E_{sy}: Elastic modulus of reinforcement, ρ_y: wall reinforcement ratio

Step1: Calculate stress and strain from forces and deformation of shear and axial spring in TVLE models



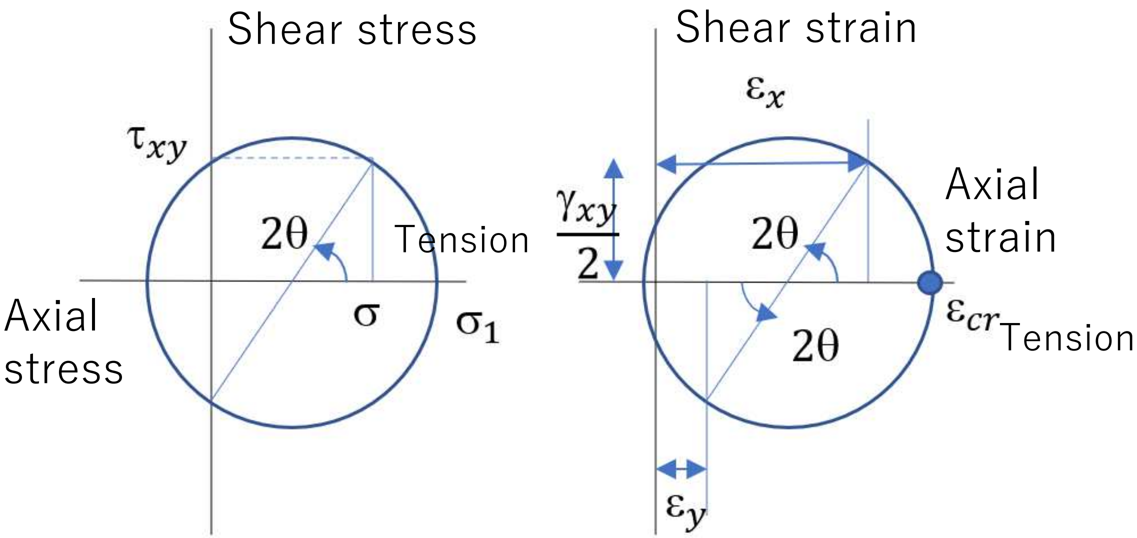
Average axial and shear stress σ, τ_{xy}

$$\sigma = \frac{N_w + N_{c1} + N_{c2}}{A_w + A_{c1} + A_{c2}} \qquad \tau_{xy} = \frac{Q_w}{A_w + A_{c1}}$$

Average axial and shear strain ε_x, γ_{xy}

$$\varepsilon_x = \frac{\delta_{axial}}{H_w} \qquad \gamma_{xy} = \frac{\delta_{shear}}{H_w}$$

Step2: Record the principal stress angle θ and the principal tensile strain ε_{cr} at the step where cracks occur.



Loading step in concrete cracking

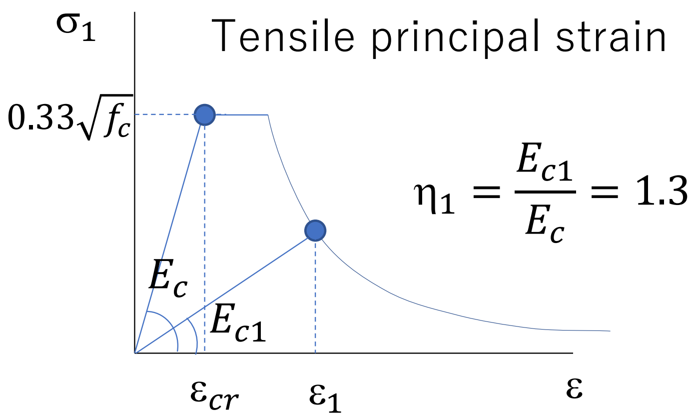
$$\sigma_1 = \frac{\sigma}{2} + \sqrt{\tau_{xy}{}^2 + \left(\frac{\sigma}{2}\right)^2} = 0.33\sqrt{f_c}$$

$$\tan 2\theta = \left(\frac{2\tau_{xy}}{\sigma}\right), \varepsilon_{cr} = \varepsilon_x - \frac{\gamma_{xy}}{2\tan 2\theta} + \frac{\gamma_{xy}}{2\sin 2\theta}$$

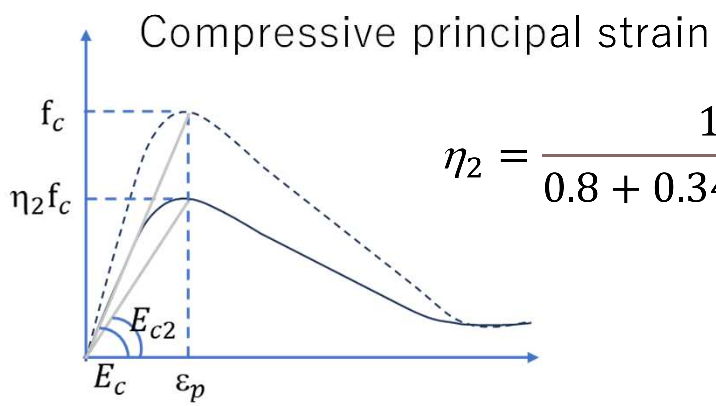
Step3: Substitute reduced concrete stiffness $\eta_1 E_{c1}$, $\eta_2 E_{c2}$ from the principal tensile strain ε_1 and angle θ after cracking step

$$\varepsilon_1 = \varepsilon_x - \frac{\gamma_{xy}}{2\tan 2\theta} + \frac{\gamma_{xy}}{2\sin 2\theta} (> \varepsilon_{cr})$$

(supposed fixed crack direction model)



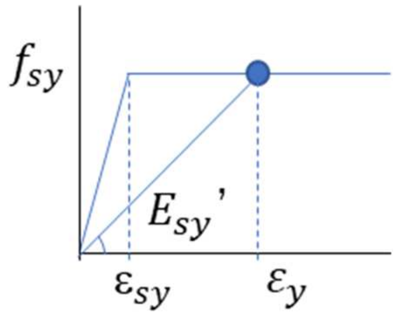
$$\eta_1 = \frac{E_{c1}}{E_c} = 1.32\left(\frac{\varepsilon_{cr}}{\varepsilon_1}\right)^{1.4} < 1$$



$$\eta_2 = \frac{1}{0.8 + 0.34(\varepsilon_1/\varepsilon_p)}$$

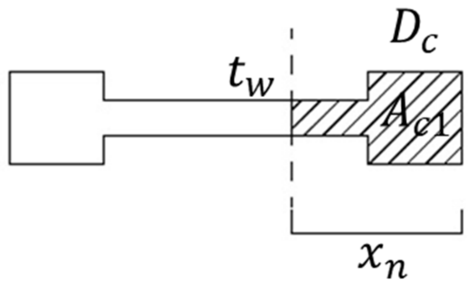
Step4: yielding of reinforcement and shear failure

If the wall reinforcement yielded, substitute the secant stiffness E_{sy}' for calculate K_s



$$\varepsilon_y = \varepsilon_x + \gamma_{xy} \frac{1}{tan2\theta} (> \varepsilon_{sy})$$

Shear failure when compressive principal stress $\sigma_1{}'$ reaches the concrete strength $\eta_2 f_c$



$$\tau_u = \frac{Q}{A_{c1} + t_w(x_n - D_c)}, \qquad \sigma_{xu} = \frac{N_{c1}}{A_{c1}}$$

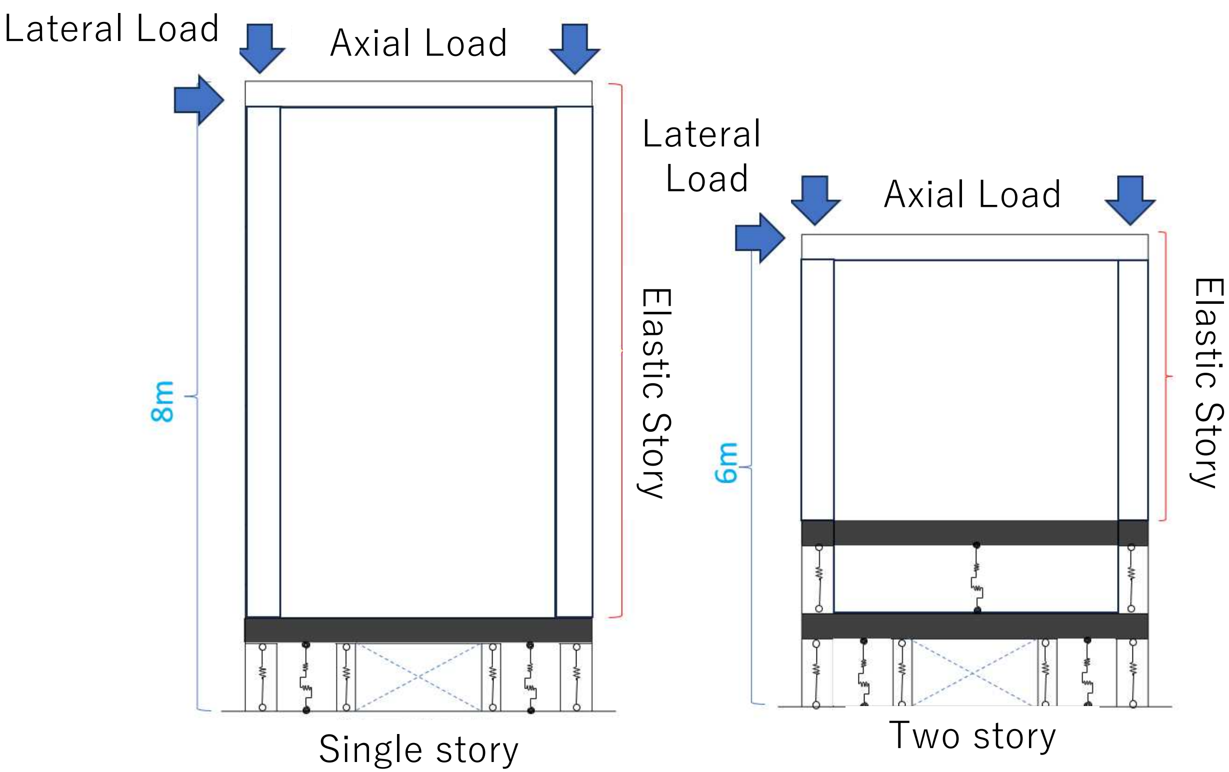
$$\sigma_1' = \frac{\sigma_{xu}}{2} + \sqrt{\left(\frac{\sigma_{xu}}{2}\right)^2 + \tau_u{}^2} > \eta_2 f_c$$

Frame Analysis (TVLE model & Axial-Shear model)

・Frame analysis with wall model of conventional Three Vertical Line Elements

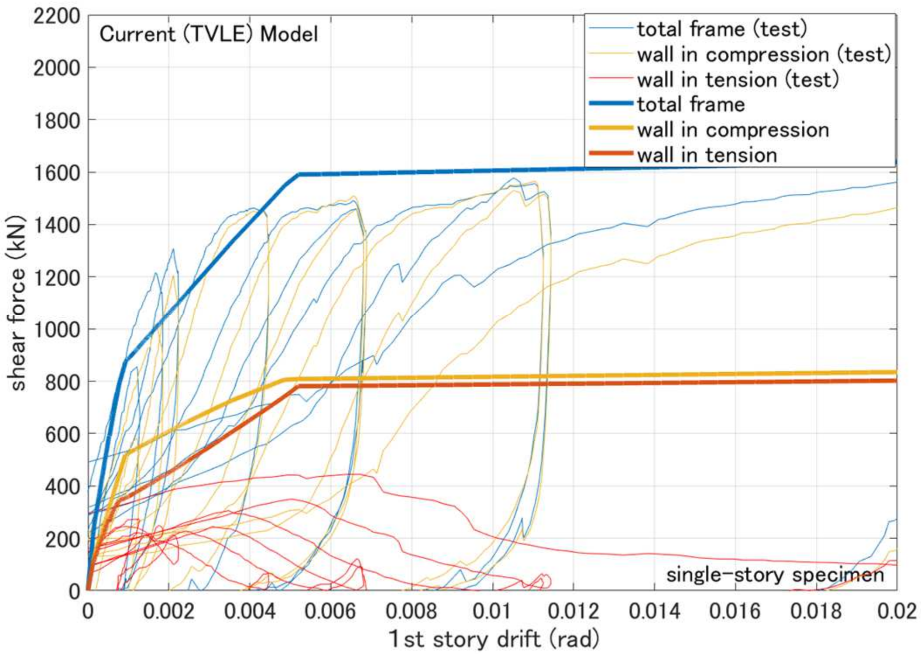
・ Frame analysis with wall model of Three Vertical Line Elements with axial shear spring

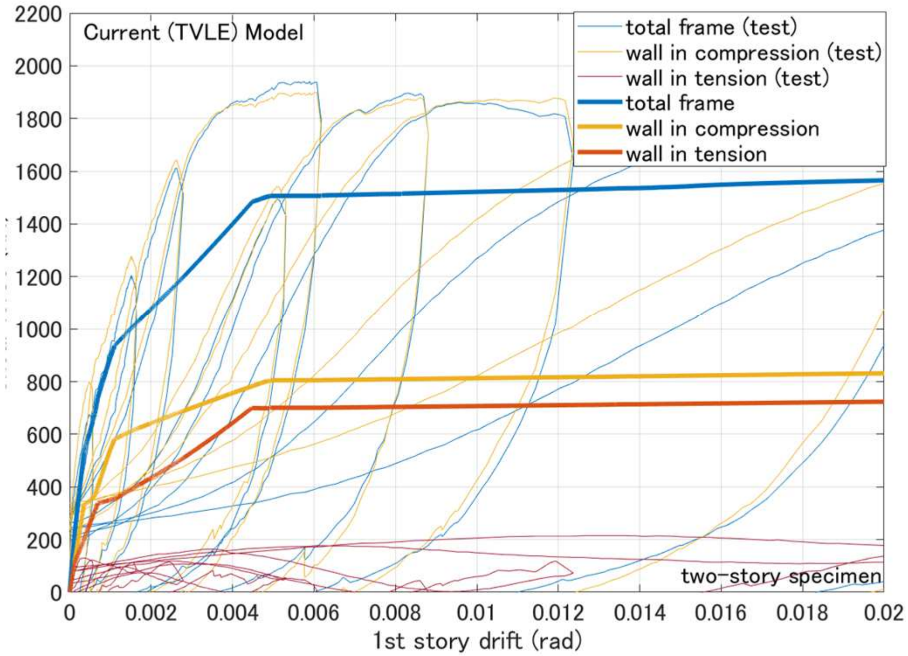
・Axial-Flexural Interaction is reflected in central flexural spring



Load-drift relation in test and analysis (Conventional)

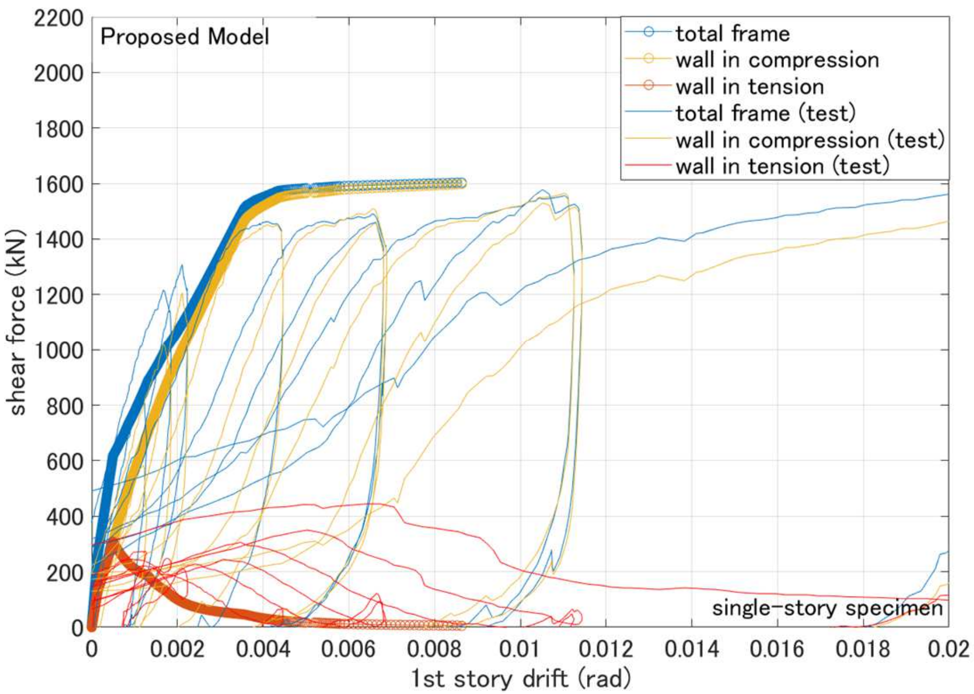
In contrast to the test results, the shear contribution is almost the same for tension and compression walls, and shear failure occurs in the model.

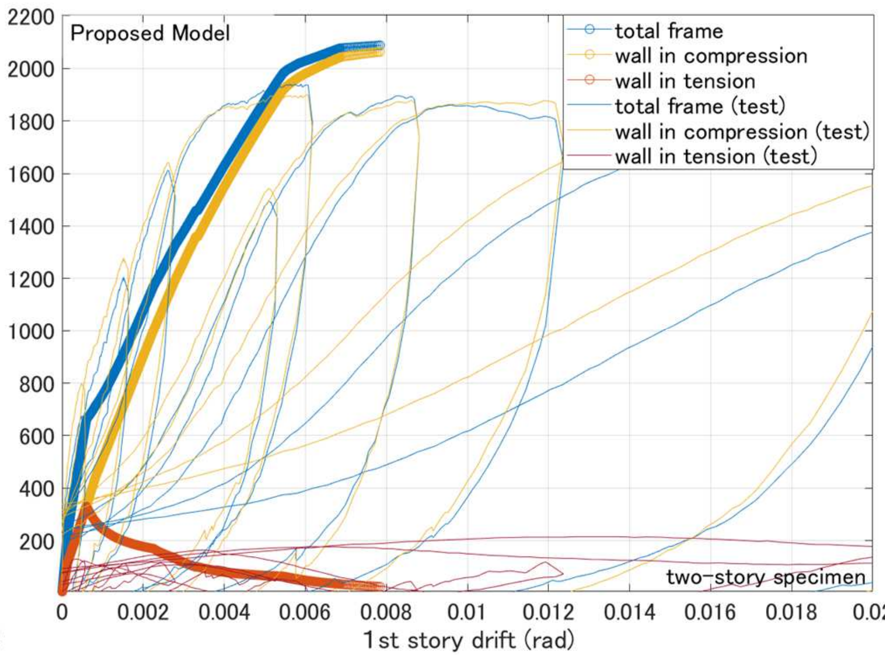




Load-drift relation in test and analysis (Proposed model)

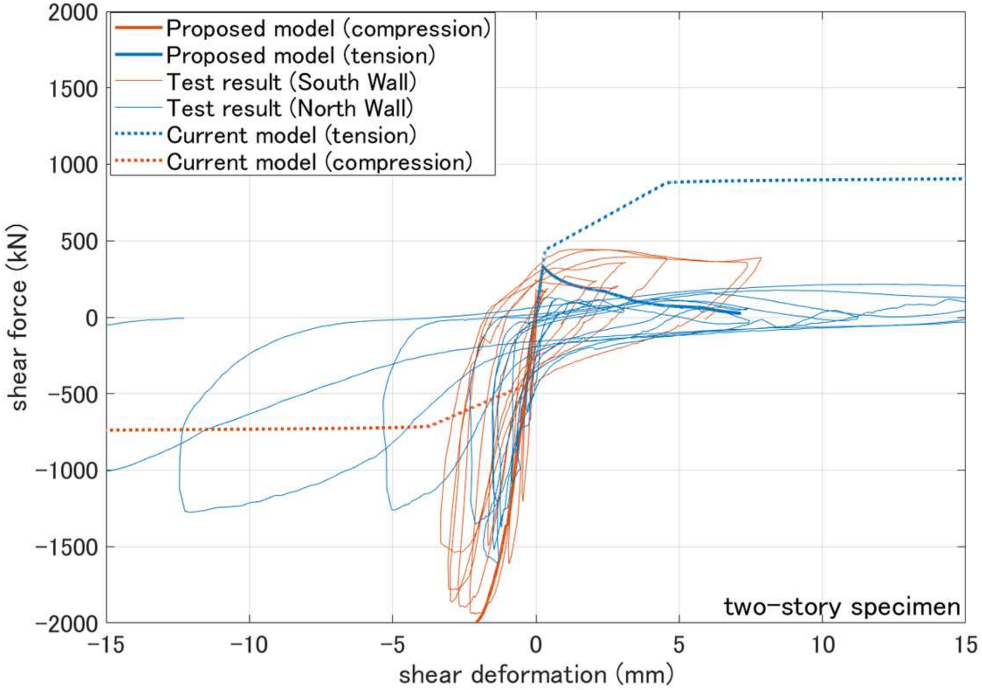
The proposed analysis method makes it available to simulate the test results in which the compression side wall carried almost all the shear force.

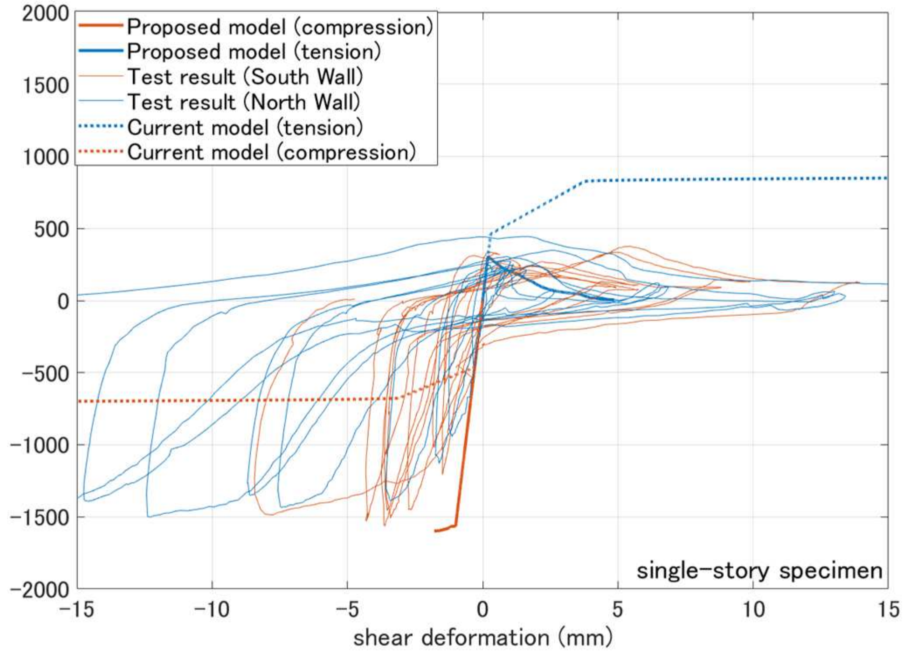




Shear force-shear drift relation in test and analysis

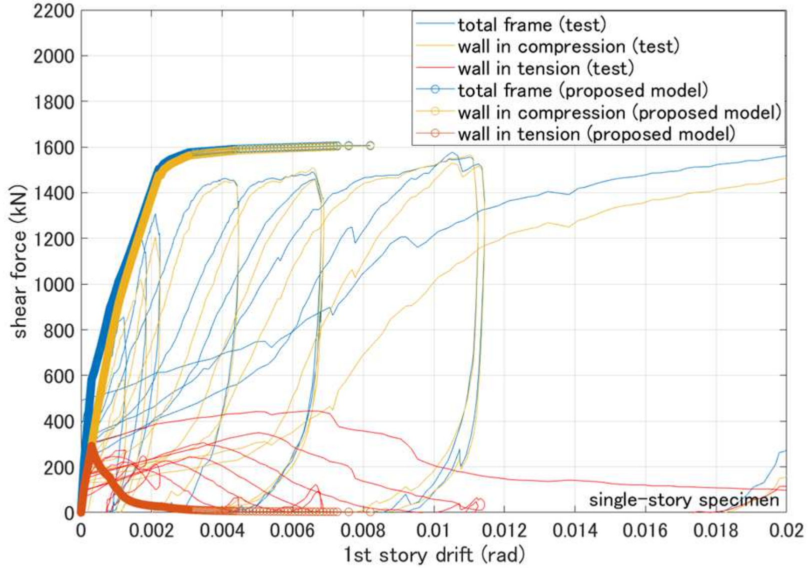
The shear stiffness does not degrade in the compression side wall,
while the shear force approaches 0 after cracks occur in the tension side wall.

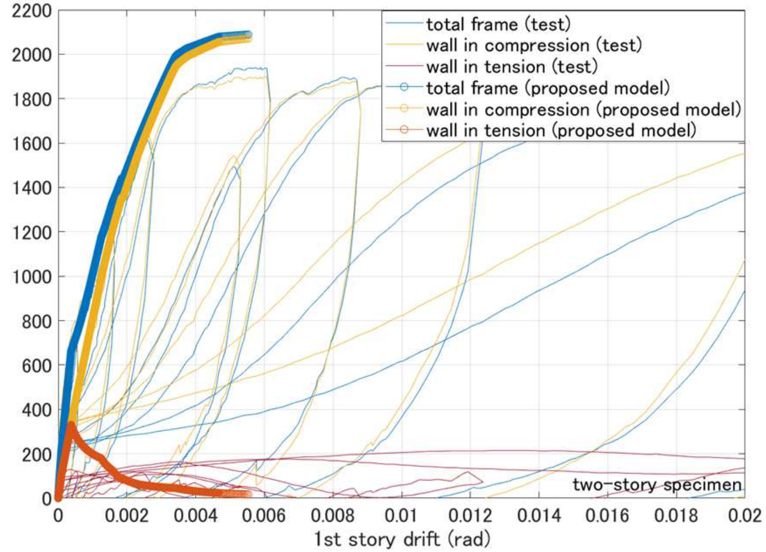


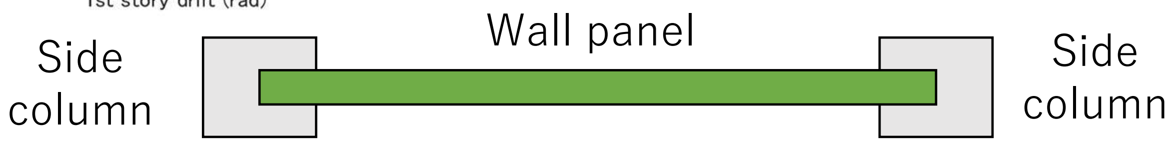


Load-drift relation in test and analysis (modified model)

Flexural stiffness is underestimated because the wall panels and columns are separated.
The cross-sectional area for flexural stiffness was assumed to be large.





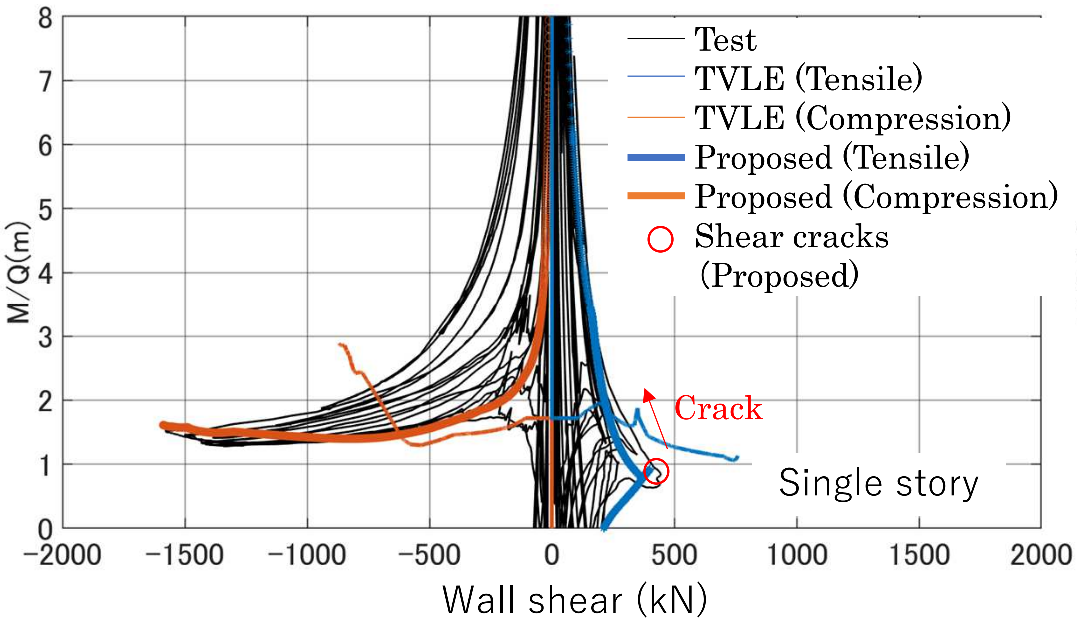


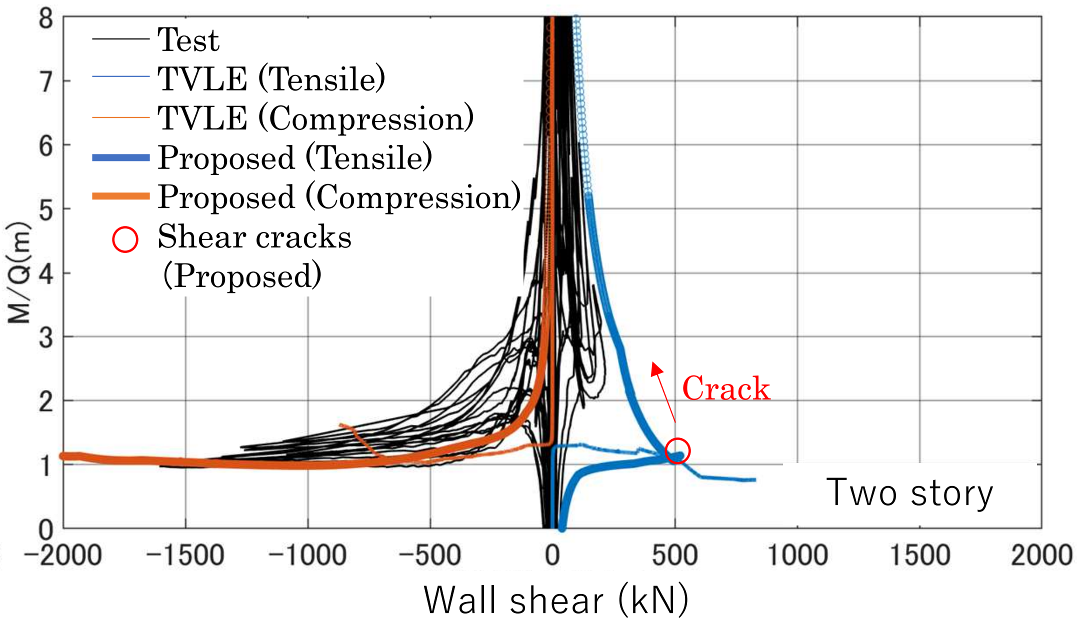
Inflection point height

・height does not change much until the ultimate state in TVLE model.

・The axial shear model represents height on the compression and tension side wall

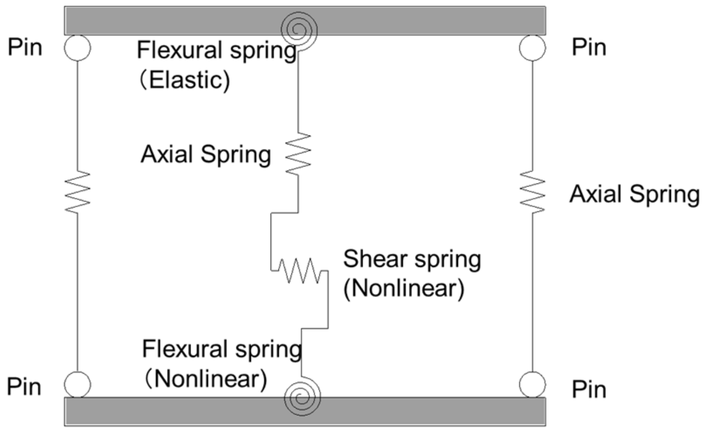
・The shear span of tensile wall is slightly underestimated at cracking in two story test specimen.
The span length increased after cracking in the axial shear model.

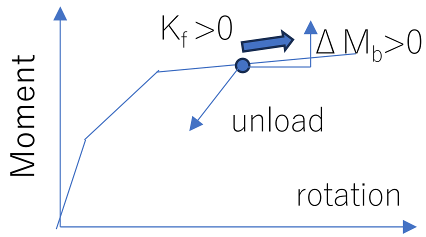


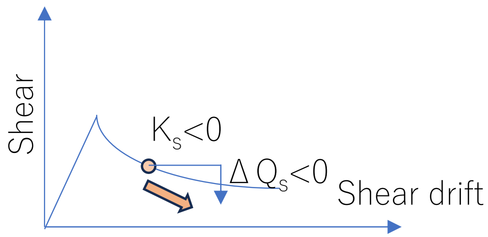


Cyclic Loading with Panel Element model

(1) Axial Shear model (Tangent Stiffness)

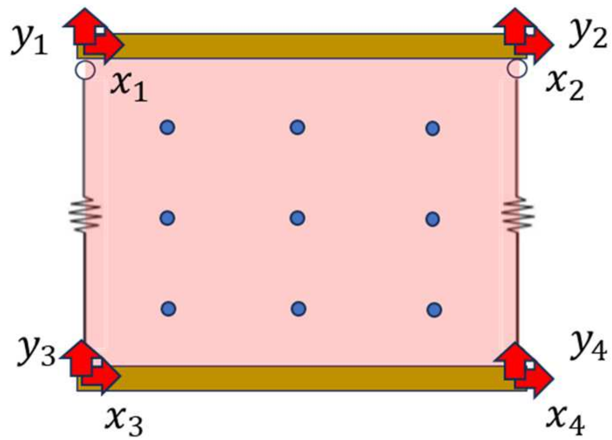






Unbalanced between shear and moment

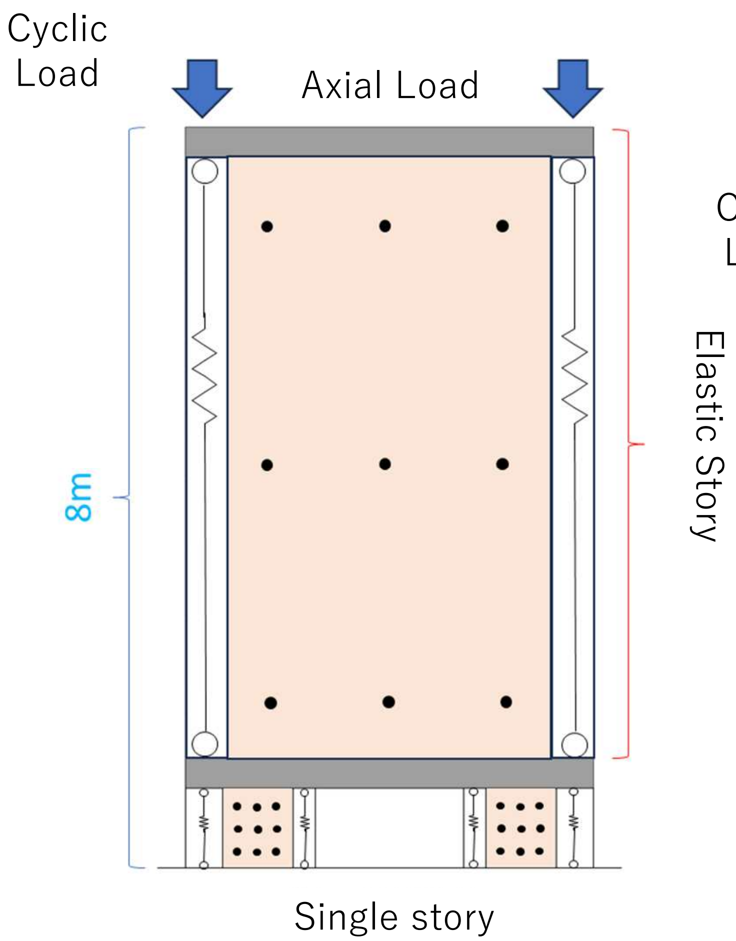
(2) Panel Element model

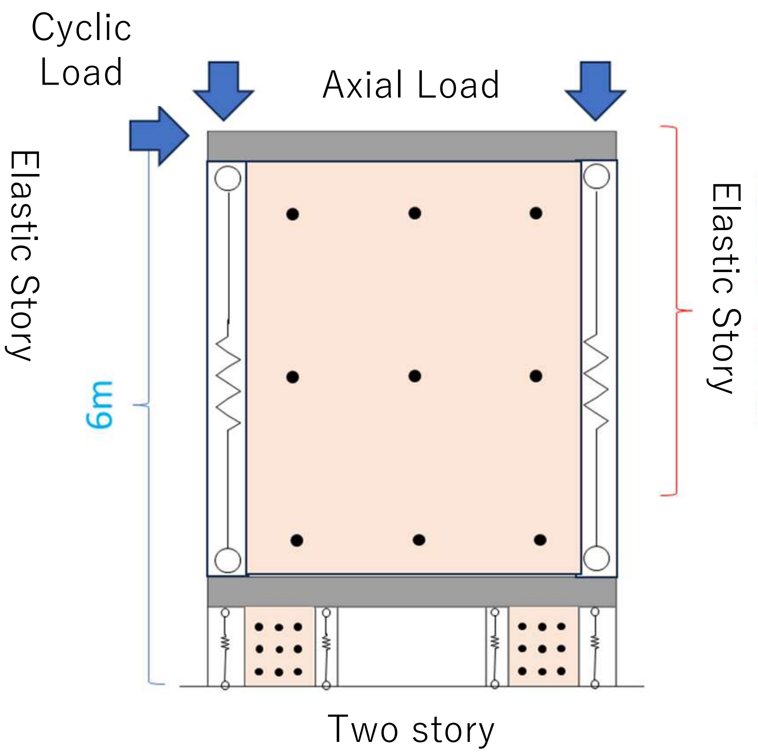


$$\begin{Bmatrix} F_{x1} \\ F_{y1} \\ F_{x2} \\ F_{y2} \\ F_{x3} \\ F_{y3} \\ F_{x4} \\ F_{y4} \end{Bmatrix} = t \iint [B]^t [D][B] \, dxdy \begin{Bmatrix} d_{x1} \\ d_{y1} \\ d_{x2} \\ d_{y2} \\ d_{x3} \\ d_{y3} \\ d_{x4} \\ d_{y4} \end{Bmatrix}$$

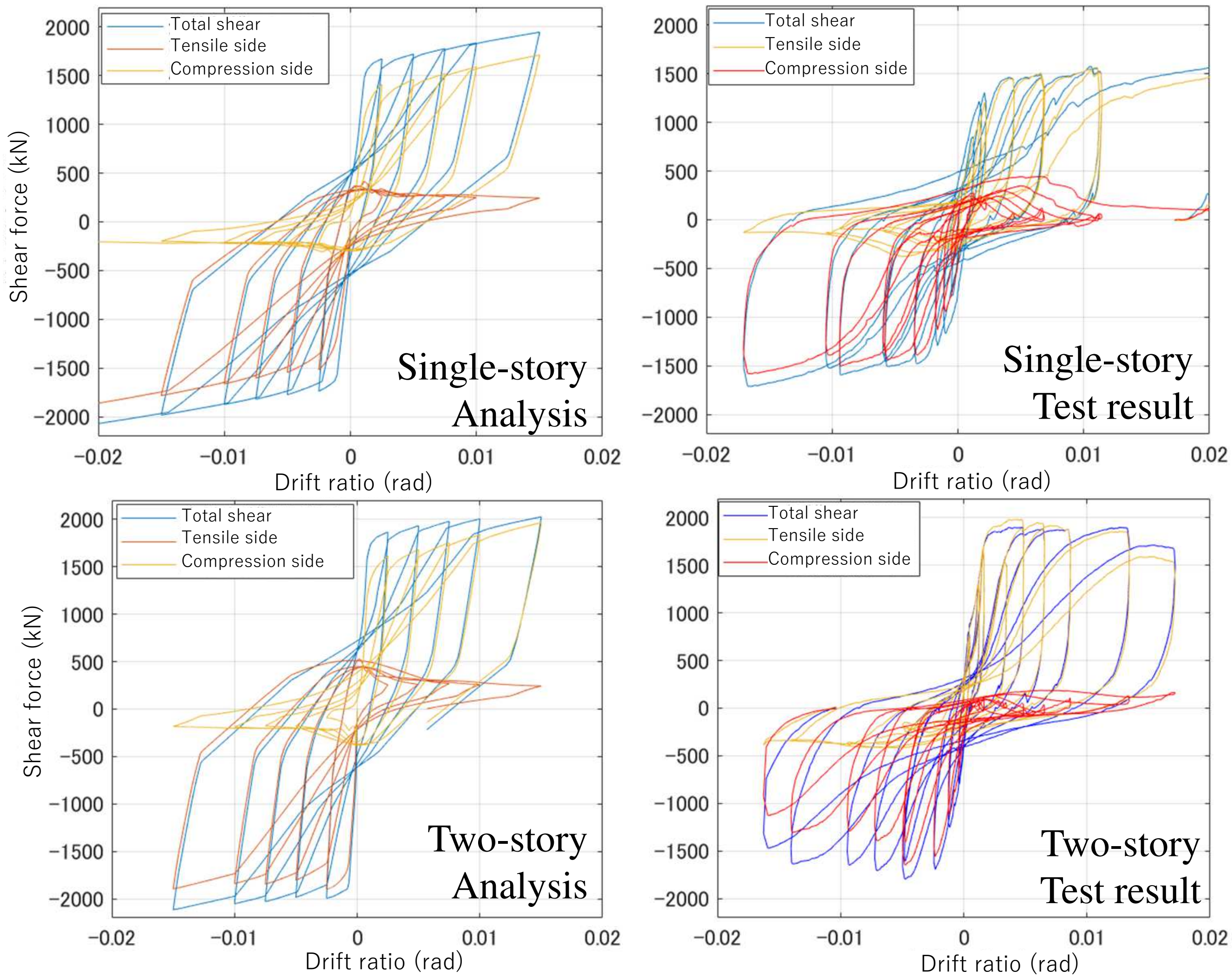
there is no need to consider balance.

Cyclic Loading with Panel Element model



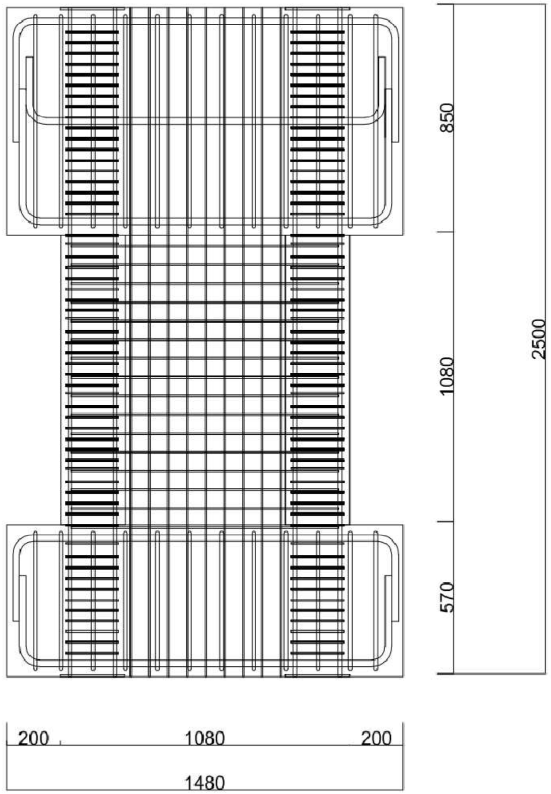


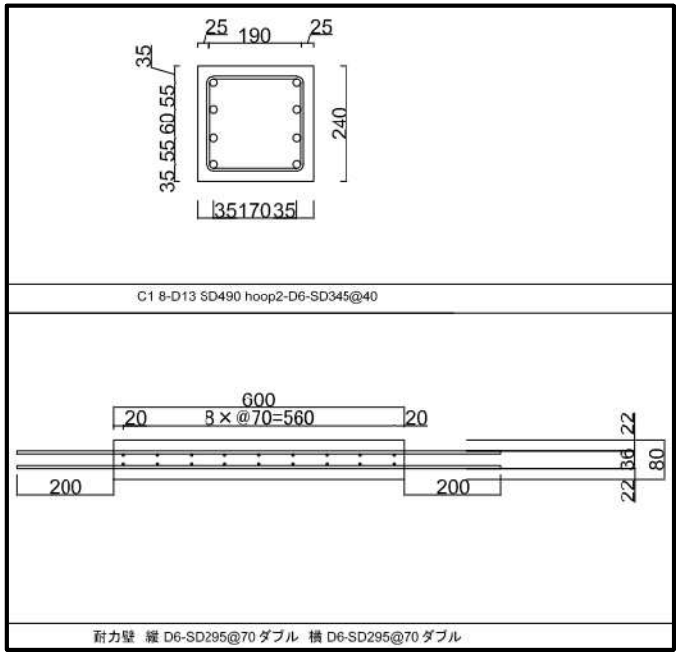
Comparison of Test result and Analysis Result



Wall Element tests with various axial load

- Loading tests on TMU 4 test specimens, Kyoto 3 test specimens
- The specimen was a 40% scale specimen with a symmetrical cross section.





Accuracy check of axial shear model

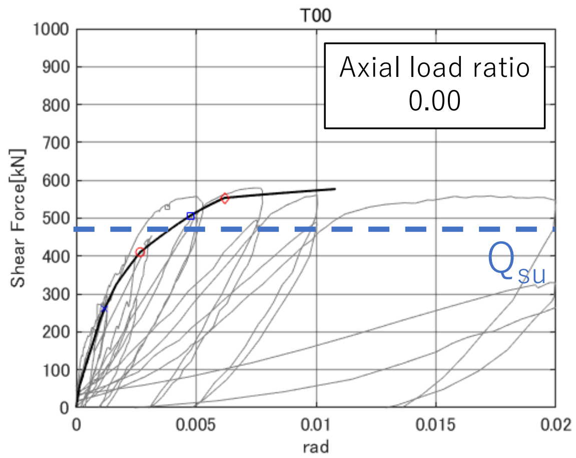
Frame Type Specimens

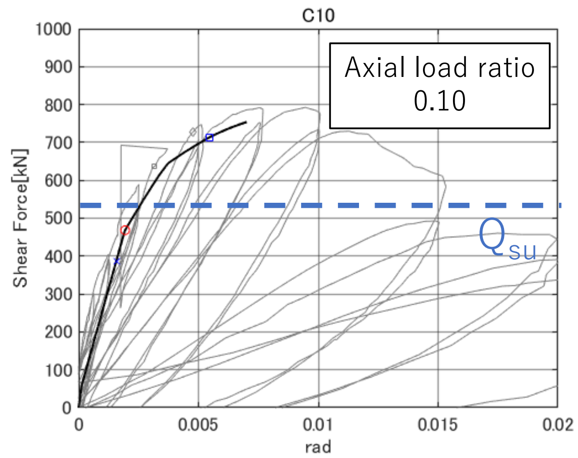
	Shear span ratio	Axial load ratio	Q_{su}/Q_{mu}
Single-story	0.79(Comp), 0.54(Tensile)	0.24(Comp)～-0.15(Tensile)	0.34(Comp), 0.46(Tensile)
Two story	0.28(Comp), 0.38 (Tensile)	0.20(Comp)～-0.12(Tensile)	0.13(Comp), 0.43(Tensile)

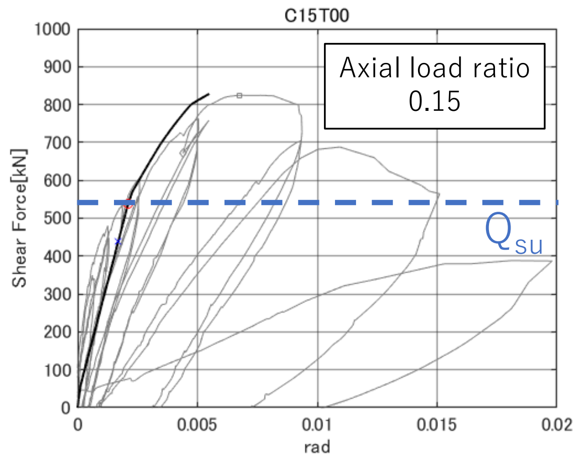
Wall Element Type Specimens

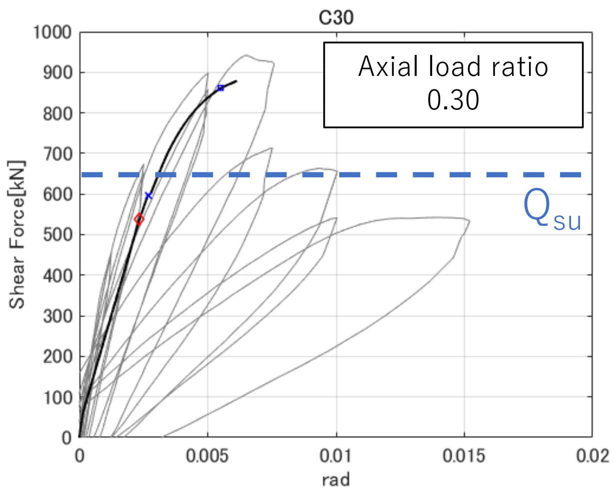
Specimen	**T00**	**C10**	**C15**	**C30**	**C40T75**
Shear span ratio	**0.86**				
Axial load ratio	**η=0**	**η=0.10**	**η=0.15**	**η=0.30**	**η=0.40 η=-0.75 (steel bar)**

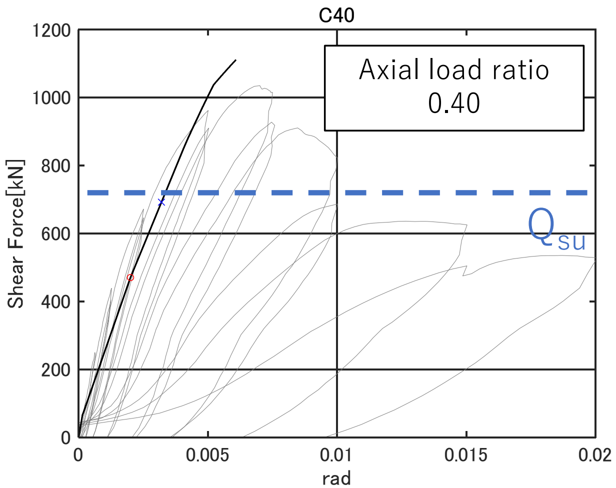
Accuracy check of axial shear model

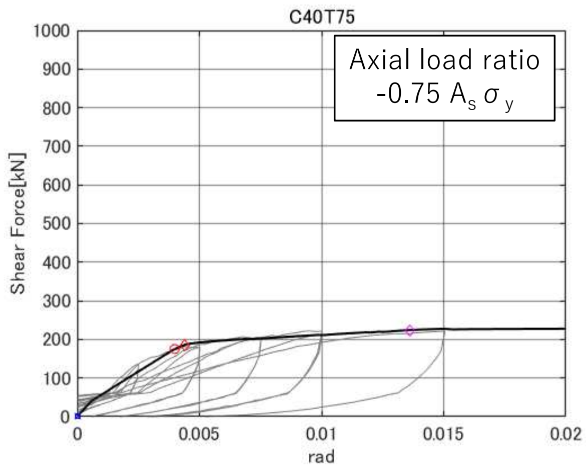






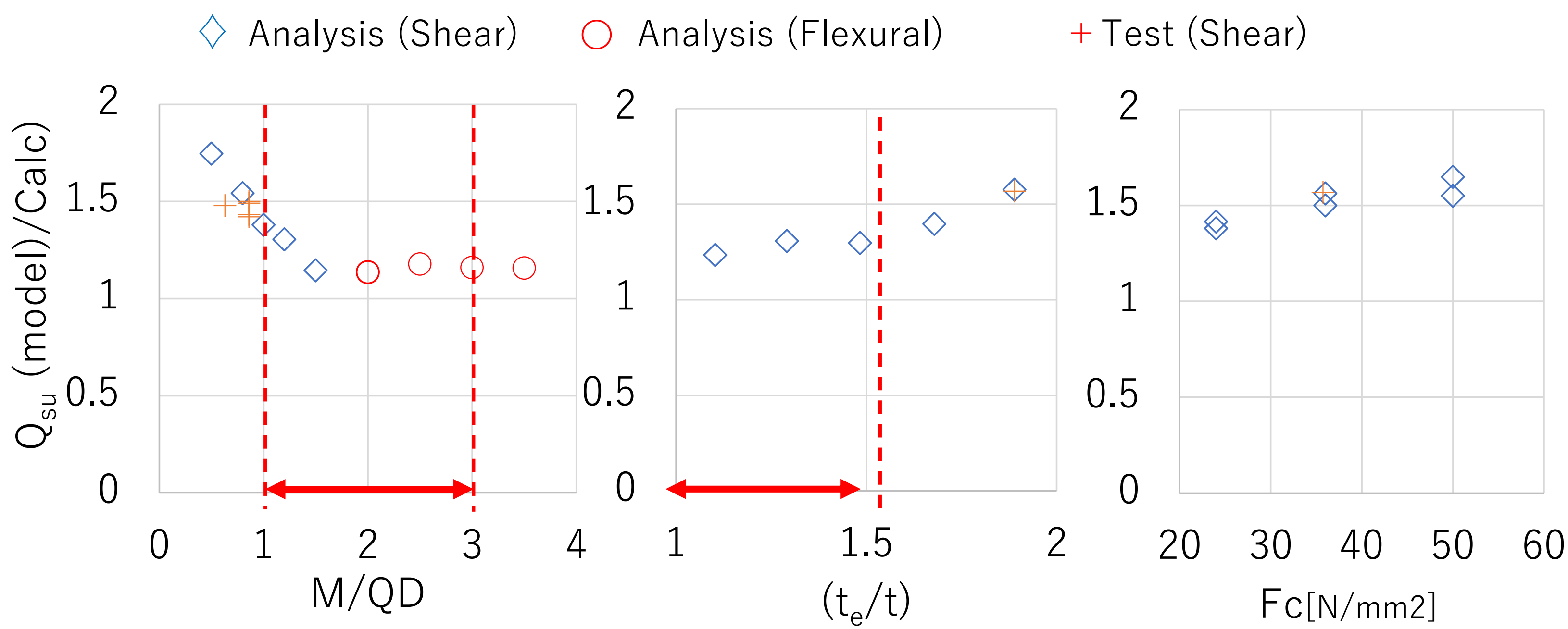






Parameter study of Axial shear model

- An analysis was carried out using wall thickness ratio, shear span, and concrete strength as parameters.
- The ratio of the design strength Q_{su} to the ultimate strength of the axial shear model was examined.
- All parameters increase linearly with respect to the design strength Q_{su}.



Concluding Remarks

- ✓ Shear contribution of the tensile wall was very small compared to that of the compressive wall, and much lower than the calculation shear strength in Japan.
- ✓ An axial shear model considering the correlation among shear stiffness, principal tensile strain and principal tensile direction was proposed based on the algorism in FEM.
- ✓ Using the proposed model, it confirmed the analytical accuracy of the load and deformation relationship of structural walls with varying axial load.
- ✓ The response to the cyclic loading was simulated with the Panel Element model.
- ✓ The accuracy of the conventional Japanese shear strength formula becomes significantly lower when the axial force, shear span, equivalent wall thickness ratio, and concrete strength exceed the assumed values in empirical formula.